Customer Based Time-to-Event Models for Cancellation Behavior: A Revenue Management Integrated Approach

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Customer Based Time-to-Event Models for Cancellation Behavior:
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To my father
and
in memory of
Axente OPRIS.

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SUMMARY

As a consequence of increased domestic competition, U.S. legacy carriers have experienced increasing pressure to incorporate customer-oriented applications into their traditional revenue management (RM) system. In this context, the main objective of this thesis is to explore the use of time-to-event methods for an important aspect of passengers' behavior, namely their cancellation behavior. Compared with similar customer-oriented applications, this research has two unique characteristics.

First, with respect to air travel behavior, it is the first study of airline passengers' cancellation behavior based on survival methods. A discrete time proportional odds model with a prospective time scale is estimated based on the occurrence of cancellations (defined as refund and exchange events) in a sample of tickets provided by the Airline Reporting Corporation (ARC). Empirical results based on 2004 data from eight domestic U.S. markets indicate cancellations are strongly influenced by both the time from ticket purchase and the time before flight departure. Higher cancellations are generally observed for recently purchased tickets, and for tickets whose associated flight departure dates are near. Cancellations are also influenced by several other covariates, including departure day of week, market, and group size.

Second, with respect to the data used, it is the first published study based on ticketing data. The use of ticketing data is motivated by the need to analyze passengers' cancellation behavior from a financial perspective. Although cancellation percentages in ticketing data are much lower (1-8%) than cancellation percentages reported in booking data (30%), they are also less volatile. In this context, we hypothesize that cancellation forecasts determined using ticketing datasets result in additional revenue when compared to current state of practice. To prove this hypothesis, this dissertation simulates and contrasts the revenue streams of a single resource capacity control under time-to-event and state of practice cancellation forecasts.

Chapter 1: MOTIVATION STATEMENT

The Airline Deregulation Act of 1978 marked the end of Civil Aeronautics Board (CAB) control over airline activities and the beginning of two decades of intense transformation. The U.S. airline deregulation was favored by (1) evidence on the viability of discount fares (*e.g.*, Skytrain's transatlantic flights), and (2) public support on the matter (support formally advocated by the head of CAB at that time Alfred Kahn). Once implemented, deregulation shifted dramatically airline industry realities both in terms of carriers' competition and passengers' opportunities.

Nowadays, legacy carriers are experiencing tremendous pressure to control costs while competing in a low-fare market that is being overtaken by low cost carriers (*e.g.*, small or regional carriers capacity grew from 150 planes in 1997 to more than 2,000 planes in 2006 (Bennett 2005)). Multiple factors have contributed to the fact that since 2001, more than 50% of the U.S. airline capacity entered into bankruptcy. While some of the factors leading to bankruptcy are well-recognized and include high fuel costs, high labor costs, and increased market penetration of low-cost carriers, other factors are less understood (*i.e.*, passengers' willingness to pay for travel or to pay for service amenities, passengers cancellation and no-show behavior, and passengers' purchasing behavior in e-markets). In this context, developing a better understanding of customer behavior and demand is seen as critical to the next generation of revenue management[1], pricing and scheduling models. Perhaps this urgency is best summarized by Suresh Achara, a director of Manugistics, who states that "there is a lot of focus on very, very sophisticated

[1] In the airline industry most of the revenue management applications are yield oriented. In this dissertation revenue management and yield management terminology will be used interchangeably.

and fancy optimization models, and that's great, but frankly, if you don't have the right demand model, if you just assume that you have the right demand value, then you're making the wrong assumption" (Achara 2005).

Currently, there is renewed interest in the airline industry in integrating discrete choice models of passenger behavior with traditional revenue management, scheduling, and other applications. This interest is renewed, not new, in the sense that as early as the 1980's several attempts were made to use discrete choice models in revenue management. However, with a few exceptions, these initial discrete choice modeling efforts were abandoned in favor of more simplistic probability models (*e.g.*, demand for booking classes on a flight arrives according to a Poisson process, cancellations are binomially distributed, etc.) and/or time-series methodologies based on historical averages (*e.g.*, the no show rate for a flight is a weighted average of no show rates for previous two months, etc.). While these probability and time-series models were easier to implement, they did not capture or explain how individual airline passengers made decisions. Moreover, many of the models currently used in practice make strong independence assumptions; *e.g.*, it is common to assume the demand associated with booking class on a flight is independent of the demand for all other booking classes on that (and surrounding) flights.

Over the last several years, these and other assumptions embedded in traditional revenue management algorithms begin to be more openly challenged (Liberman and Yechiali 1978; Ratliff 1998; Oliveira 2003; Boyd 2004; Boyd and Kallesen 2004; Hornick 2004; Talluri and van Ryzin 2004b; Dunleavy and Westermann 2005), forcing a re-examination of how one can model individual airline passengers' behavior using discrete choice or other models grounded in behavioral theory. Recent work using discrete choice methods for revenue management include that by Garrow and Koppelman (2004a; 2004b) for no show applications, Ratliff (1998) for demand unconstraining and recapture applications, and Talluri and van Ryzin (2004b) who explore the use of a simple multinomial logit (MNL) embedded in a optimization model to determine seat allocation levels.

In the general context of revenue management practice, this research is motivated by three aspects. First, from a methodological perspective, this dissertation introduces customer-based models to the cancellation forecasting practice. Indeed, despite the fact that small improvements in forecasting accuracy of demand models can translate to millions of dollars in annual revenue for an airline (Neuling, Riedel et al. 2004), the cancellation models used in practice are still fairly simplistic. Based on a review of the academic literature and practitioner conference proceedings, it was determined that most, if not all, cancellation models are based on historical averages that consider the influence of a small number of covariates associated with an itinerary (*e.g.,* day of week, departure time, origin and destination, etc.) or with a booking (*e.g.*, booking class and group size).

Second, from a data perspective, this dissertation updates cancellation models state-of-practice with empirical findings derived from ticketing data. Indeed, given cancellation forecasting was one of the earliest revenue management practices, adopted properties associated with the cancellation process (*i.e.*, memoryless, group independence) were empirically derived using pre-deregulation data (Thompson 1961; Martinez and Sanchez. 1970). While tractable, these findings are questionable in the context of an increasingly commoditized air-travel service and different reservation rules.

Third, from a business perspective, this thesis adheres to the "competitive airline markets research" (Belobaba and Wilson 1997) introduced by Boeing Commercial Airplanes (BCA). Specifically, BCA has been engaged in a research effort to advance its models of passenger behavior. These models are a central part of the tools used by its marketing department to help potential airline customers estimate how much market share and revenue can be gained via the introduction of new service and equipment in a market.

One of the core components of the passenger behavior models under development is the Universal Market Simulator (UMS) shown in Figure 1-1.

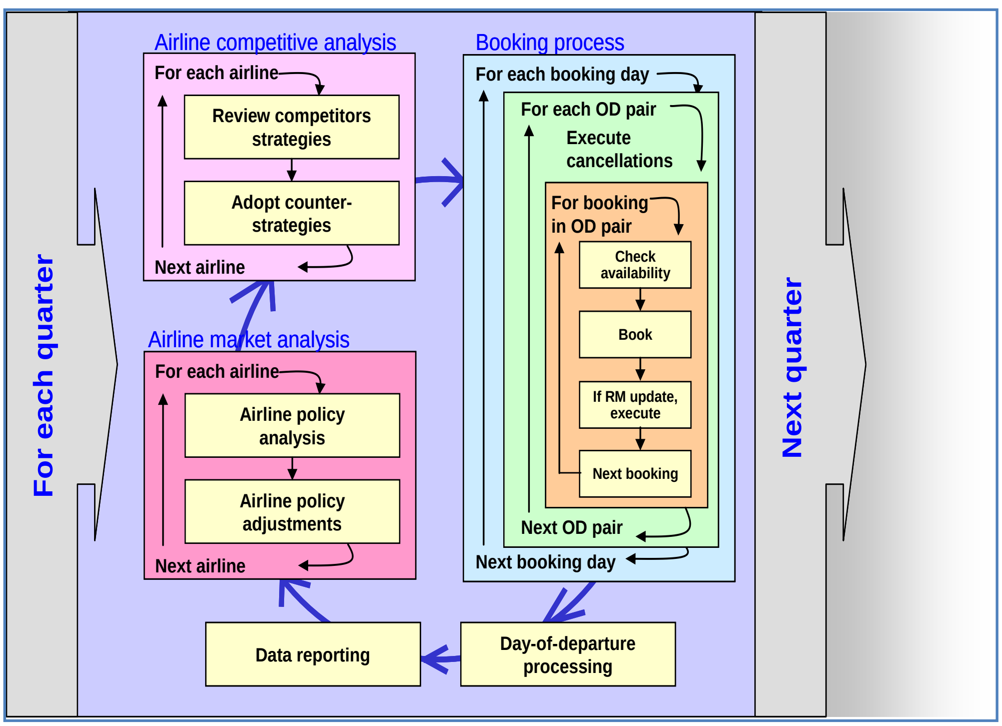

Figure 1-1: Boeing's Universal Market Simulator

The UMS is a Monte Carlo micro-simulation of airline revenue generation whose primary output is the revenue to an airline that results from the individual choices of thousands of passengers moving over a world-wide airline network. The UMS uses several models to represent different aspects of passenger behavior and airline competitive responses including models for synthetic population generation, induced demand, booking and ticketing curves, ticket cancellations, passenger itinerary choice, and airline revenue management models (Parker, Lonsdale et al. 2005).

To conclude, as a novelty to previous research in the field of cancellation models this research focuses on quantifying the intensity of the cancellation process with respect to departure and/or issue date. Its methodological details and findings come to support the transition from an inventory driven (*i.e.* traditional) revenue management to a customer centric revenue management. Primarily designed for the general UMS context, this dissertation is the first to use a data source that permits the analysis of the passengers' cancellation behavior from a financial perspective. As such, its applicability extends to ticketing clearinghouses, airlines or travel agents in need of a better control of their revenue stream.

Following the research motivation, this dissertation has six chapters. Chapter 2 identifies main characteristics of current state of practice of cancellation models and defines the scope of research. Chapter 3 frames the literature review of cancellation models in the more general context of yield management. Chapter 4 selects and describes the data source used in current dissertation. Chapter 5 formulates the methodological details of the two areas of research: (1) time-to-event analysis of cancellation process and, (2) revenue management implementation of time-to-event forecasts. Chapter 6 presents research results. Finally, Chapter 7 points out the contributions of thisdissertation and identifies areas of future research.

Chapter 2: PROBLEM DEFINITION

This dissertation focuses on updating the state of practice of cancellation models in two areas: (1) the type of data used for analysis, and, (2) the way in which the intensity of the cancellation process is estimated.

With respect to the first category, this dissertation is the first to explore properties of the intensity of cancellation process using ticketing data. While traditional cancellation models mandate the use of booking information, revenue estimation procedures recommend the use of ticketing information. Since the intensity of the cancellation process is analyzed in the context of unearned revenue or Air Traffic Liability (ATL), findings from this thesis can be used to adjust revenue opportunity measures associated with inventory and overbooking control policies.

The second area in which this thesis adds to the state of practice is the way in which cancellation behavior is modeled. While the importance of forecasting timing effects to overboooking algorithms has been overwhelmingly acknowledged by the yield management community[2], there have been no studies which focus on analyzing the true transitional properties of the cancellation process. Typically, the intensity of the cancellation process is assumed to be stationary (Thompson 1961) or forwardly anchored (*i.e.*, depends only on departure date). In contrast, this dissertation studies the combined effects of the issue date and departure date on the conditional intensity of the cancellation process.

[2] Used primarily for overbooking methods cancellation estimators influence capacity allocation decisions as well.

To underline differences between this approach and previous work in this area, the following sections present a short description of the state of practice for cancellation models, define methodological updates, and discusses areas of applicability.

2.1. Cancellation Models – The State of Practice

Airlines use seat inventory control to decide how many seats (associated with a set of prices) to make available for sale to customers. However, since not all customers who request seats actually travel, airlines overbook to reduce the expected number of empty seats on flights when there is demand for those seats. Although, the importance of optimal seat inventory control and overbooking decisions in obtaining revenue gains is well-known (Smith, Leimkuhler et al. 1992), theoretical formulations in which the two yield management decisions are addressed simultaneously (*i.e.*, the general yield management problem) are scarce. Also, due to the complexity of legacy carriers (LC) inventory operations, these few exceptions are generally[3] not implementable.

Challenges associated with the implementation of exact solutions for the general yield management problem in practice are overwhelmingly acknowledged by the revenue management community. Philips (2005) states that "the combined overbooking and capacity allocation problem is extremely difficult to solve in general" because of "the fact that not only are different booking classes likely to have different fares, they are also likely to have different cancellation and no-show rates." Talluri and Van Ryzin (2004a) point out the difficulties associated with overbooking algorithms in the presence of customer class mix noting that such approaches need to keep "track of the inventory of

[3] To best of the author's knowledge, the only exception is the EMSR heuristic (Belobaba 1989) where the seat allocation controls are adjusted with overbooking factors.

each class as a separate state variable and then make overbooking decisions based on this complete vector of state variables."

In the general framework of airline inventory operations, forecasts of cancellation and no-show rates are used to set up controls for overbooking levels, *i.e.*, the number of seats authorized for sale that exceed the capacity of the flight (see Figure 2-1). The difference between cancellation and no show forecast models relates to when the airline knows passengers do not intend to travel. Cancellation models predict how many passengers inform the airline they do not intend to travel prior to the departure of their flights while no show models estimate the number of remaining booked passengers, *i.e.*, passengers who have not cancelled, but fail to show for their flights.

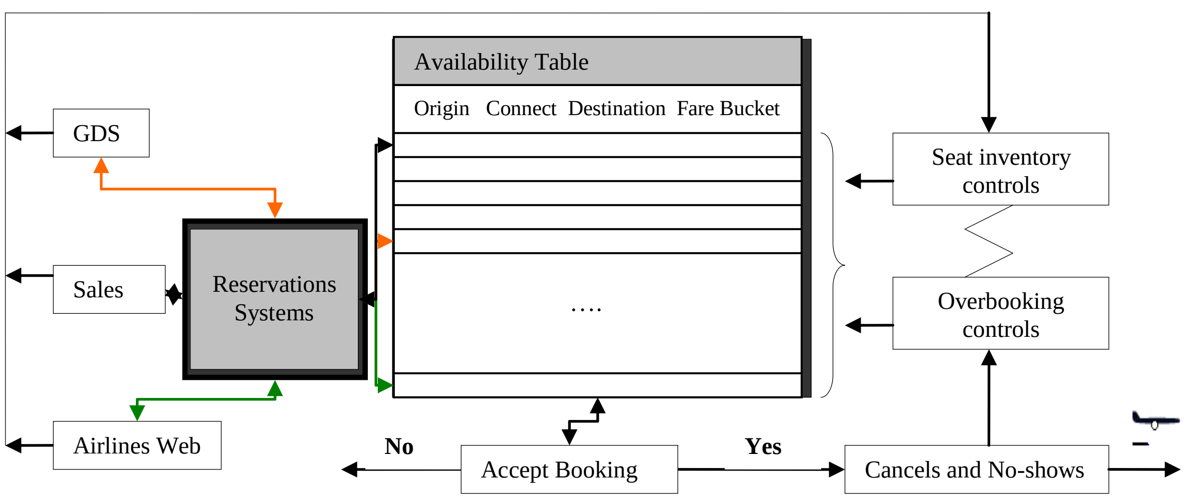

Figure 2-1: Legacy Airlines Inventory Operations

As seen in Figure 2-1, legacy carriers' inventory is made available through several distribution channels: Global Distribution Systems (GDS[4]), travel agencies (direct or internet sales), and airline proprietary distribution systems (typically the airline web-site). As a forefront of the revenue management "black-box" reservation systems are a

[4] GDS's have the competitive advantage of seamless availability (*i.e.*, high look-to-book ratio from different sources in real-time)

collection of integrated solutions specifically designed to communicate and update the availability of airlines inventory in real-time. Accept/reject decisions are highly automatized, request queries being directed towards an availability table which consists of a collection of market/fare buckets with the latest information on seat-inventory and overbooking controls at the time of booking. Finally, as an off-line mechanism of revenue management forecasting and optimization procedures, seat-inventory and overbooking controls take into account fluctuations in characteristics of demand and are periodically updated.

In the context of inventory operations, the state of practice for cancellation forecasts is to estimate the intensity of the cancellation process with respect to forward time periods. Although different types of cancellation models are discussed in the literature, for the purpose of this discussion two categories[5] are worth mentioning: static (proportions) or dynamic (rates).

Static cancellation models estimate the probability of current bookings surviving until departure date, *i.e.*, survival proportion. In contrast, dynamic cancellation models estimate the probability of current bookings surviving until the next period, *i.e.*, survival rate (see Figure 2-2). In practice, estimators of these two probabilities are determined as non-parametric ratios: proportions – as a ratio of show demand to current bookings (Talluri and Van Ryzin 2004a) and rates - as ratio of total cancellation in a time period to the number of bookings at the beginning of that time period (Subramanian, Stidham et al. 1999).

[5] Other cancellation models not reported in the literature are also used in practice. For example, some cancellation models estimate (1) the number of *currently active* bookings at time t that will survive until departure, and (2) the number of *future bookings* that will arrive (and survive) between time t and flight departure.

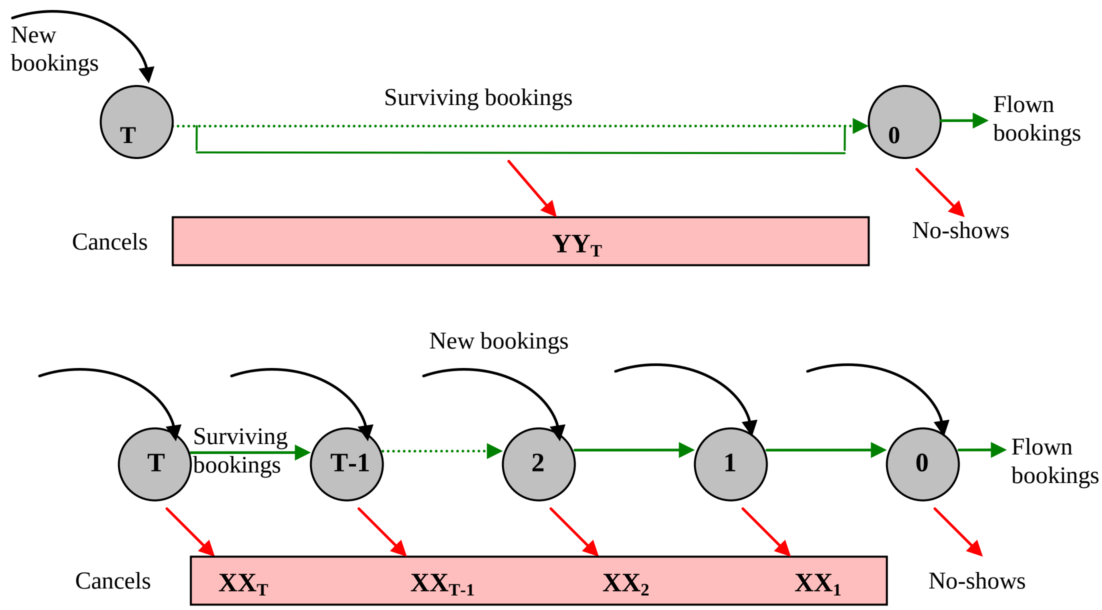

Figure 2-2: Cancellation Models: (a) Surviving Proportions; (b) Survival Rates

For the first category of estimators, Talluri and van Ryzin (2004a) note that it is common to use the probability of bookings surviving until departure (q_t) as an input to a binomial distribution which is used to set up overbooking controls. For the second category, Subramanian, et al. (1999) use non-parametric estimators of cancellation rates as descriptors of transitional probabilities for a mixed dynamic programming (MDP) formulation of a general yield management problem.

The main problem associated with incorporating more realistic behavioral assumptions into cancellations estimation procedures is the fact that they need to be implemented in existing revenue management systems. For example, the intensity of the cancellation process is determined by the frequency of cancellations from bookings on hand and does not take into account the effect of time of booking (*i.e.*, is anchored in future times of booking horizon). Nevertheless, this memoryless assumption makes dynamic formulations more tractable. Since the cancellation process is assumed to be stationary (*i.e.*, independent of the time of booking) changes in cancellation probability are straightforward to incorporate into the value function of a MDP formulation[6].

2.2. Methodological Updates

When compared to current industry practice of determining "cancellation rates" estimates differ in two aspects: (1) the way in which population at risk of cancelling is considered and, (2) the influence of time from booking on the cancellation process. Industry cancellation rates use as the population at risk the total number of bookings on hand and assume that the cancellation process is independent of the time from booking (memoryless property). In contrast, time-to-event cancellation rates use the number of current and future bookings "alive" at a certain day from issue as population at risk and assume cancellation process depends on the time from booking.

If we define $di_{,(i=1\ldots3)}$ - demand with time of booking i , $cij_{,(i=1\ldots3,\ j=1\ldots3)}$ - cancellations at time period j for bookings with time of booking i, and $sj_{(j=1\ldots3)}$ – number of bookings lost in time periods j, the set of Equations 2-1 and 2-2 present the sample estimators of cancellation rates ($ri_{,(i=0\ldots3)}$) and cancellation hazards ($hi_{,(i=0\ldots3)}$). The set of Equations 2-1 points out that cancellation rates are determined using only past bookings (*i.e.*, time of booking is before current time). In contrast, cancellation hazards are determined using past and future bookings.

Figure 2-3: Cancellation Rates and Cancellation Hazards – Sample Estimators

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[6] The probability of a cancellation is directly linked to the state variable vector, *i.e.*, $q_n(x)$ – probability of a cancellation in period n given that the current number of bookings on hand is x

$$r_3 = \frac{c_{33}}{d_3}$$
$$r_2 = \frac{c_{32} + c_{22}}{d_3 - c_{33} + d_2}$$
$$r_1 = \frac{c_{31} + c_{21} + c_{11}}{d_3 - c_{33} - c_{32} + d_2 - c_{22} + d_1}$$
$$h_0 = \frac{c_{30} + c_{20} + c_{10}}{d_3 - c_{33} - c_{32} - c_{31} + d_2 - c_{22} - c_{21} + d_1 - c_{11}} \quad (2.1)$$

$$h_3 = \frac{c_{33} + c_{22} + c_{11}}{d_3 + d_2 + d_1}$$
$$h_2 = \frac{c_{32} + c_{21} + c_{10}}{d_3 - c_{33} + d_2 - c_{22} + d_1 - c_{11}}$$
$$h_1 = \frac{c_{31} + c_{20}}{d_3 - c_{33} - c_{32} + d_2 - c_{22} - c_{21}}$$
$$h_0 = \frac{c_{30}}{d_3 - c_{33} - c_{32} - c_{31}} \quad (2.2)$$

The second difference between the two categories of estimates relates to the way in which timing effects are considered. Estimates of cancellation hazards depend on days from issue (*i.e.*, backwardly anchored). In contrast estimates of cancellation rates depend on days from departure (*i.e.*, forwardly anchored). Also, since the life-span of bookings is known (equal with time of booking minus departure date), the estimates of cancellation hazards which take into account only the effect of days from issue (*i.e.*, survival time) are not uniquely defined.

To account for differential chances of being at risk of cancelling, the influence of days from departure has to be considered as well. Studied simultaneously, effects of these two covariates (days from issue and days from departure) allow the analysis of the intensity of the cancelation process from a new perspective: new bookings with respect to future periods in the booking horizon. In this case, days from departure (DFD) acts as a "treatment variable" which, in combination with days from issue, permits the estimation of cancellation proportions of new bookings for different times of booking.

presents differences between the two approaches using sample estimators of cancellation rates and hazards and a booking horizon equal to three time periods. If we define $d_{i,(i=1...3)}$ - demand with time of booking i , $c_{ij,(i=1...3,\ j=1...3)}$ -cancellations at time period j for bookings with time of booking i, and $s_{j\ (j=1...3)}$ – number of bookings lost in time periods j, the set of Equations 2-1 and 2-2 present the sample estimators of cancellation rates ($r_{i,(}$

$_{i=0...3)}$) and cancellation hazards ($h_{i,(\ i=0...3)}$). The set of Equations 2-1 points out that cancellation rates are determined using only past bookings (*i.e.*, time of booking is before current time). In contrast, cancellation hazards are determined using past and future bookings.

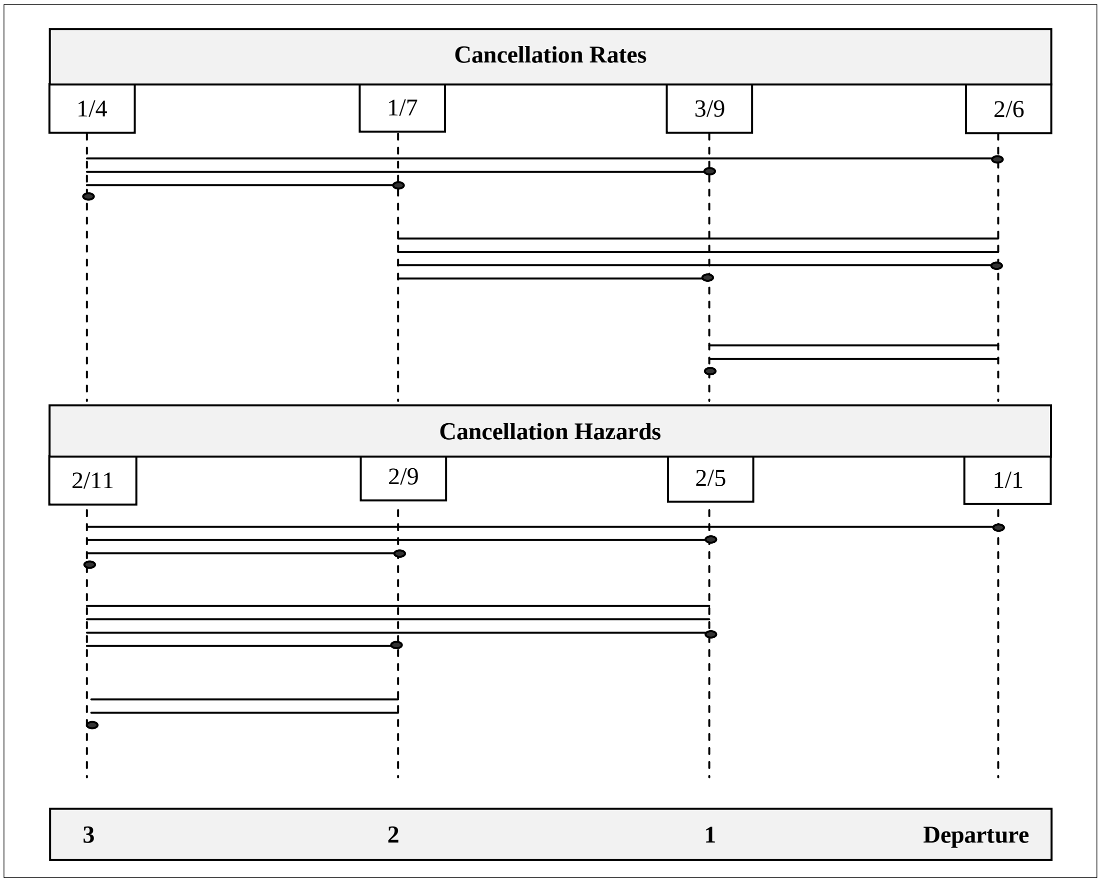

Figure 2-3: Cancellation Rates and Cancellation Hazards – Sample Estimators

$$r_3 = \frac{c_{33}}{d_3} \qquad h_3 = \frac{c_{33} + c_{22} + c_{11}}{d_3 + d_2 + d_1}$$

$$r_2 = \frac{c_{32} + c_{22}}{d_3 - c_{33} + d_2} \qquad h_2 = \frac{c_{32} + c_{21} + c_{10}}{d_3 - c_{33} + d_2 - c_{22} + d_1 - c_{11}}$$

$$r_1 = \frac{c_{31} + c_{21} + c_{11}}{d_3 - c_{33} - c_{32} + d_2 - c_{22} + d_1} \qquad h_1 = \frac{c_{31} + c_{20}}{d_3 - c_{33} - c_{32} + d_2 - c_{22} - c_{21}}$$

$$h_0 = \frac{c_{30} + c_{20} + c_{10}}{d_3 - c_{33} - c_{32} - c_{31} + d_2 - c_{22} - c_{21} + d_1 - c_{11}} \qquad h_0 = \frac{c_{30}}{d_3 - c_{33} - c_{32} - c_{31}}$$

(2.1) (2.2)

The second difference between the two categories of estimates relates to the way in which timing effects are considered. Estimates of cancellation hazards depend on days from issue (*i.e.*, backwardly anchored). In contrast estimates of cancellation rates depend on days from departure (*i.e.*, forwardly anchored). Also, since the life-span of bookings is known (equal with time of booking minus departure date), the estimates of cancellation hazards which take into account only the effect of days from issue (*i.e.*, survival time) are not uniquely defined.

To account for differential chances of being at risk of cancelling, the influence of days from departure has to be considered as well. Studied simultaneously, effects of these two covariates (days from issue and days from departure) allow the analysis of the intensity of the cancelation process from a new perspective: new bookings with respect to future periods in the booking horizon. In this case, days from departure (DFD) acts as a "treatment variable" which, in combination with days from issue, permits the estimation of cancellation proportions of new bookings for different times of booking.

2.3. Area of Applicability for Time-to-Event Forecasts

Using ticketing datasets, one of the objectives of this dissertation is to explore the intensity of the cancellation process from a more complete transitional perspective. All tickets are "born" on the issue date and have a predetermined and known life-span. Time-to-event rates result as conditional intensities of a cancellation event happening, intensities are adjusted to take into account differential chances of being at risk in the first place. In contrast to the current state of practice which estimates cancellation rates/proportions for bookings on hand and assumes the memoryless property, estimates focus on determining the combined effect of days-from-issue and days from departure for new bookings.

Our hypothesis is that in the early stages of a ticket life-span, time from issue is one of the most important drivers of passengers' (*i.e.*, tickets) cancellation behavior. As tickets become older, the effect of days from issue decreases and the effect of days from departure increases. Provided this holds true, this dissertation quantifies the extent to which a time-to-event forecast of cancellation rates/ proportions applied to a general revenue management context results in revenue gains. To assess the value of this forecasting exercise, the impact of the "hazard-determined" overbooking controls on the revenue stream is estimated.

To determine the revenue impact of methodological updates this dissertation simulates and compares revenue streams for a single resource capacity control under time-to-event cancellation forecasts and state of practice forecasts. As an alternative to the current state of practice of estimating the intensity of the cancellation process as a function of bookings on hand and days from departure, we propose estimating the intensity of the cancellation process as a function of new bookings, days from issue and days from departure.

The second objective of this dissertation is to prove that traditional yield management methods can be improved by a better understanding of cancellation timing effects. Similar in spirit with discrete choice revenue management, this dissertation focuses on incorporating ***survival analysis results*** into the state of practice of yield management. In this context the main limitation of this research resides in the fact that is does not provide an exact solution to the "time-to-event yield management problem."

There are several reasons why this dissertation does not incorporate a general non-stationary dynamic programming formulation. First, as already referenced, adding a new dimension to the combined overbooking and capacity allocation problem will exponentially increase computational requirements of revenue management systems. Second, since the primary focus of current thesis is on forecasting cancellations using a novel approach, *i.e.*, time-to-event models, we wanted to make sure that this type of forecast is likely to generate revenue increases for the worst case scenario[7]. Finally, given limitations of available data sources (*i.e.*, missing control and socio-demographic information) we wanted to keep the set of assumptions which might influence the reliability of revenue estimation procedures to a minimum.

To better understand the challenges associated with the general yield management problem and the way in which cancellation forecasts help solve it, the next chapter presents a literature review.

[7] Revenue increases of exact algorithms are higher when compared with their heuristics counterparts. In this context current dissertation aims to provide a lower bound on the possible revenue increase generated by time-to-event forecasts of cancellations

Chapter 3: LITERATURE REVIEW

Despite its importance, the literature on cancellation models is relatively scarce. With the exceptions of a few stand-alone empirical studies, methodological advances related to the cancellation area are generally found in yield management papers. In this context, our literature review focuses on describing cancellation models in the context of yield management practice. Section 3.1 presents an historical perspective on the yield management practice and points out differences in the airline industry before and after deregulation. In the context of the general yield management problem, Section 3.2 identifies cancellation research advances. Section 3.3 concludes the chapter by summarizing main findings of the literature review.

3.1. The Airline Deregulation Act – Promoter of the Yield Management Practice

Despite 20th century technological advances, the airline industry did not present a competitive alternative to well-established long-distance modes (trains and ocean liners) until the introduction of commercial jet aircrafts (1958). With the rapid increase of air traffic the need for regulatory structures capable to address conflicting needs of passengers, airlines and governmental structures becomes stringent. In the U.S., the Civil Aeronautics Board (CAB), a governmental entity created by the Civil Aeronautic Act of 1938, was the first to promote commercial air travel and to protect interests of air passengers. CAB legislative activities were complimented by the Federal Aviation Administration (FAA) which handled regulations related to airline maintenance and safety and the Air Traffic Conference (ATC) which handled procedures related to interlining[8].

With CAB supervising airlines' new routes and ticket prices, true competition among carriers was inhibited and incentives to address customer needs diminished (Bennett 2005). As a result, by the end of 1960s, public criticisms directed towards the CAB "regulatory failure" started to be more vehement and pressures to liberalize airline industry increased. Although public expectations were partially fulfilled[9] during the Ford administration, the deregulation movement reached critical momentum during Carter administration with the inauguration of Alfred Kahn[10] as chairman of the Civil Aeronautics Board (CAB). In a series of legislative acts culminating with the Airline Deregulation Act of 1978, Kahn liberalized airlines entry and pricing structure and abolished the CAB regulatory authority.

The impact of deregulation on the airline industry was fourfold. First, in what Poole and Butler (1999) identified as a three-wave process the market penetration of airline service increased dramatically. At the expense of multiple stops, the new hub-and-spoke route system promoted by legacy carriers permitted passengers' access to multiple destinations. The negative effects of traffic increase at hubs were capitalized by low cost carriers (LCC) which offered alternative point-to-point routes at lower fares then the legacy arriers (LC) carriers. Finally, regional jets carriers provided access to airline service to passengers from smaller cities.

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[8] Interlining refers to transfer baggage and reservations across multiple carriers. Today, interlining covers regulations with respect to partnership between airlines and travel agents, global airline alliances and code-sharing agreements.
[9] Air cargo is deregulated and discount fares are permitted for the first time.
[10] An economics professor, from Cornell University, Alfred Kahn was well known for his critiques directed towards the traditional CAB regulation.

Second, domestic competition between LC and LCC airlines increased. Although initial attempts such as PEOPLE Express seemed to indicate that in the long run low-cost carriers could not compete with legacy carriers, LCC such as Southwest, JetBlue and AirTran proved the contrary. Their success was not only due to the "low-fare, no frills and point-to-point service" mantra, but also resulted from clear operational advantages. For example, Southwest avoided hub competition by serving secondary airports (*e.g.*, Providence, Rhode Island instead of Boston, Massachusetts) and promoted an employee oriented culture. Similarly, in early 1999 JetBlue started with startup capitalization ($130 million dollars) and preferential access to 75 slots at John F. Kennedy (JFK) International.

Third, to better address the challenges of the newly created competitive market, airlines invested heavily in their distribution systems. The practice of "sell and record and wait lists" (Beckman 1958) was gradually replaced by mainframes capable of handling seat inventory distribution in real-time. Started as Computer Reservation Systems (CRS) and later transformed into Global Distribution Systems (GDS), the new distribution systems ensured quick and reliable access to unbiased travel content (*i.e.*, available inventory and fares across multiple providers) to travel agents all around the world.

Fourth, with carriers understanding that differences in various streams of demand can provide "opportunities to adjust for imperfections in the airline's schedule design" (Belobaba 1987), premises of revenue management practice were initiated. Started as a broad concept with the objective of "maximizing passenger revenue by selling the right seats to the right customers at the right time" (American Airlines 2005) the revenue management practice was gradually refined to a collection of demand models, forecasting methods, and optimization algorithms which addressed two categories of tactical demand-management decisions[11] - price and quantity based (Talluri and Van Ryzin 2004a). In the general RM practice, methodological advances of both categories of decisions were equally important. In the airline RM practice most revenue gains were attributed to methodologies developed to address quantity based decisions such as **seat inventory control** and **overbooking**.

As a "tactical component of the revenue management that is entirely under the control of each individual airline" (Belobaba 1989), seat inventory control focused on determining capacity allocation policies which maximizes revenue or yield across carrier's network. Although, in its most general form, a seat inventory control can be schematized as a collection of A1-B1-C1-D1-E(1/2/…i)-F4-G3-H3-I2-J2-K2-L5-M2-N3 set of elements[12], previous research in this field addressed only simplifying versions (Beckman 1958; Littlewood 1972; Pfeifer 1989; Curry 1990; Smith, Leimkuhler et al. 1992; Weatherford and Bodily 1992; Brumelle and McGill 1993; Lee and Hersh 1993). In contrast to seat inventory control, overbooking focused on reducing the expected number of empty seats due to cancellations or no-shows. Despite being the oldest of the RM practices, overbooking success was hard to predict given its clandestine[13] start (Rothstein 1985). Still, after Ralph Nader won a law-suit against Allegany Airlines the importance of overbooking in maintaining satisfactory yield levels[14] was openly admitted

[11] Structural decisions such as which selling format to use or how to bundle services/ products are also part of the RM practice. Still, due to their strategic character they are less frequent.
[12] Defined according to the taxonomy of perishable assets (Weatherford and Bodily 1992).
[13] At start the practice of overbooking was not acknowledged by airlines. Rothstein talks in great detail about this and arguments it with his personal experience within the industry
[14] Empirical reports indicate that 40% to 50% of reservations result either in cancellations or no-shows - Thomson (1961).

by carriers and favorable premises for future methodological advances were set (Rothstein 1971; Shifler and Yardi 1975; Bodily and Pfeifer 1992; Smith, Leimkuhler et al. 1992; Chatwin 1998; Karaesmen and Van Ryzin 2004).

Due to the complexity of the yield management environment[15], most of the previous RM research addressed seat inventory control and overbooking problems separately. Despite this methodological divide, their commonality was overwhelmingly acknowledged and research efforts to address the general yield management problem increased over the time. While a comprehensive literature review on the airlines yield management practice is beyond the scope of this thesis, identifying areas of applicability for current cancellations models proves to be extremely relevant. In the context of the general yield management problem, the following section presents cancellation and no-shows methodological advances and addresses their limitations.

3.2. The General Yield Management Problem and Cancellation Research

With cancellation percentages averaging 30% (Talluri and Van Ryzin 2004a) and exhibiting high volatility across the booking horizon, the cancellation effect cannot be ignored when forecasting the future net demand or when deciding on how to allocate inventory or set up overbooking controls. Despite its importance, the cancellation literature is relatively scarce and empirical examples of implemented cancellation models are few. Understandably, this situation is maintained by airlines which are "reluctant to share information about their forecasting methodologies because their revenue management activities are so heavily dependent on accurate forecasting" (Talluri and

[15] "Yield management problem is best described as a nonlinear, stochastic, mixed-integer mathematical program that requires data, such as passenger demand, cancellations, and other estimates of passenger behavior, that are subject to frequent changes" (Smith, Leimkuhler and Darrow, 1992).

Van Ryzin 1999). Still, cancellation research pertains to several revenue management (RM) practices. The literature on overbooking, combined overbooking and seat inventory control, and demand forecasting can be used to establish the state of practice for cancellations models.

After deregulation, airlines introduce hub-and-spoke operations and discount fare classes to optimize network operations and increase revenue. As a consequence, the complexity of yield management (YM) environment increased tremendously.

Although, most of RM applications address capacity control and overbooking in isolation, the "problems of optimizing demand mix and volume are quite related" (Talluri and Van Ryzin 2004a). Despite their connection, exact solutions for the combined capacity control and overbooking problem are difficult to implement in practice. For example, in dynamic programming algorithms the presence of cancellations and no-shows complicates the computation of booking limits to a point where exact methods, although theoretically tractable, become computationally infeasible[16] for industry applications. As an alternative, the industry practice associates a set of simplifying assumptions[17] to heuristics and addresses overbooking and capacity allocation decisions in a sequential manner.

Since the majority of YM literature addressed seat allocation or overbooking decisions in isolation, the research directed towards "the general" YM approach are

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[16] "To solve the system-wide yield management problem would require approximately 250 million decisions variables" Smith, B., J. Leimkuhler, et al. (1992). "Yield management at American Airlines." Interfaces **22**(1): 8-31.

[17] The inventory assumptions of a flight are traffic related, *i.e.*, whether to consider multiple-flight connecting markets versus single-flight markets and policy related, *i.e.*, whether to consider refunds/penalties for no-shows and cancellations or not. In contrast, the demand assumptions of a flight are process related, *i.e.*, whether arrival and cancellation patterns are time-dependent or not and distribution related, *i.e.*, whether there exists stochastic dependence between full fare and discount customers or not

scarce. In an effort to simplify methodological challenges associated with the general YM problem, Belobaba (1989) proposes a version of the Expected Marginal Seat Revenue (EMSR) heuristic in which seat inventory control rules[18] are periodically adjusted by overbooking factors.

Subramanian, Stidham et al. (1999) suggest a more sophisticated approach for associating the seat allocation problem to a dynamic programming model which allows for cancellations, no-shows and overbooking. Their experimental findings are particularly important in quantifying the impact of exact overbooking solutions to airlines' revenue streams. Using a small example (*i.e.*, capacity equal with 4 an overbooking pad equal with 2) and class depended cancellation and no-show rates, Subramanian et al. compute percentages of revenue "sacrificed" for different cancellations scenarios.

Their results indicate that incorporating class-dependent cancellations into exact solutions for yield management problem can result in a 9.39% revenue increase. Interestingly, the revenue impact varies significantly with the way in which cancellations are incorporated. If only fares are adjusted (*i.e.*, "effects of cancellations on state variables are omitted from MDP optimality equations") or all cancellation rates are assumed to match cancellation rates of highest fare class, incorporating cancellation effect results in revenue decreases of 2.28% and 17.34% when compared with the no-cancellation case. If cancellation rates are introduced as averages of cancellation rates of two or more classes or are determined as class-independent cancellation rates which

[18] In the EMSR framework the seat inventory control decision was defined as accept or not accept the discount unit. The decision rule was nothing more but an adaptation of the popular news-vendor model and it said that discount units should be accepted as long as the probability of a spill (*i.e.* the probability that the current discount demand plus the full-price demand will exceed capacity) is less or equal then the ratio of the discount fare to the full-fare.

"approximate as closely as possible" the exact solutions, incorporating cancellation effects results in revenue increases of 6.34% and 8.92% when compared with the no-cancellation case.

Although notable exceptions from the overall state of practice of addressing the two categories of controls in isolation, the two papers only reinforce the idea that cancellation forecasts are process rather than customer oriented. In that perspective, the following paragraphs track methodological and conceptual advances of cancellation models during pre-deregulation and post-deregulation eras.

Before pre-deregulation, with airlines activity regulated and monitored by Civil Aeronautics Board (CAB), the incentive to invest in sophisticated inventory control systems was limited. As a consequence, reservation systems were highly simplified with reservation agents maintaining "sell and record" and "wait" lists on the available space. Accumulated sales were monitored by airlines headquarters, which upon observing a certain level of occupancy, would issue a stop sales message to reservation agents. Between the time of a stop sales message and until departure, monitoring activities were deferred to reservation agents which, in cases of unexpected loss of passengers (*e.g.*, failure to purchase ticket or cancellations), could accept passengers from wait lists. Finally, a "departure control" list containing updates on losses such as late cancellations, no-shows or misconnections and adds such as errors, standbys or removals was provided a few hours before departure to the airport stations (Beckman 1958).

As a result of reservation operations being highly fragmented, the percentage of no-shows and cancellations was extremely high. With one out of ten passenger not showing up (CAB 1961) and 40% of reservations being cancelled (Thompson 1961) economic challenges of the "sell and record" and "wait" list system were more than obvious. Well aware of this problem, the Civil Aeronautics Board tried to curb the no-show phenomenon and proposed as solution the no-show penalty (CAB 1961). Although CAB initiative has "partially met its objectives" (CAB 1964), difficulties in identifying true no-shows[19] and apprehensiveness of airline executives in implementing the penalty made this success short lived (Rothstein 1985).

With respect to methodological advances of cancellation models during pre-deregulation, the work of Beckman (1958), Thompson (1961) and Littlewood (1972) are worth noticing. In the context of optimal communication between "space control" and travel agents, Beckman (1958) determines the optimal sales values (*i.e.*, the overbooking limits) by fitting a gamma distribution to "the demand and loss distributions."

Exploring challenges associated with yield values in the pre-deregulation reservation control, Thompson (1961) is the first to propose a non-parametric estimation of cancellation rates and to test the validity of a binomial distribution for cancellations (*i.e.*, given a fixed number of confirmed bookings - N, cancellations are assumed to be Bernoulli trials with a probability of success - p). Another important assumption introduced by Thompson is the stationarity of the cancellation process. Similar to a Poisson process, the intensity of the cancellation process, *i.e.*, the probability of an event happening, is considered constant for "all intervals of the same length and independent of the past history of the system" (Thompson 1961). Thompson states the independence of cancellations across different bookings and points out possible departures from this assumption in the case of groups.

[19] Passengers with late connecting flights were mistakenly identified as authentic no-shows

Finally, in the first paper to introduce the newsvendor model to the overbooking practice, Littlewood (1972) forecasts passenger cancellation rates using a simple exponential smoothing procedure. While his empirical findings state that both seasonality and trend effects of the day of the week are negligible, the accuracy of forecast is mentioned as a possible source of concern.

After deregulation to efficiently accommodate the hub-and-spoke operations and increase coordination with the newly created global distribution systems (GDS)[20], legacy carriers' research departments redesigned their seat inventory systems. The old "sell and record" and "wait" list routine was replaced by sophisticated operation research models capable of addressing complex demand-management decisions in real time. While this revamp of distribution operation resulted in significant revenue gains[21], reports on cancellations and no-show percentages remained high. For example, one of the early promoters of YM practice, American Airlines quotes that "on average, about half of all reservations made for a flight are cancelled or become no-shows. American estimates that about 15 percent of seats on sold-out flights would be unused if reservation sales were limited to aircraft capacity" (Smith, Leimkuhler et al. 1992).

Despite the high percentage of cancellations and no-shows, the interest in cancellation methodological updates remains secondary to airline researchers and practitioners. The pre-deregulation empirical findings of Thomson (1961) and Martinez and Sanchez (1970) are frequently referenced. Starting in the late 1990's this situation changed as pre-deregulation findings related to cancellation were contested by the

[20] Nowadays, at the expense of booking fee supported by participating airlines four major GDS systems (Amadeus, Galileo, Sabre and Wordspan) ensure a complete automation of the reservation process for travel agents.
[21] Typically in the airline industry revenue management systems have been credited with gains of 2 to 5 % (Belobaba and Wilson, 1997)

empirical evidence provided by two AGIFORS[22] presentations (Westerhof 1997; Chatterjee 2001).

Using a sample data from KLM sample data, Westerhof computes the probabilities $\boldsymbol{p_T}$ of bookings surviving from one booking period T to the next booking period T-1 to prove that the memoryless property of cancellation probabilities is violated. His findings on cancellation rates/ proportions are reinforced by Chatterjee who points out that another important assumption on cancellation probabilities, the independence assumption, does not hold for groups.

3.3. Overview of the Literature Review

To conclude, the literature review of cancellation models reveals that the most of the studies in this field dates prior to deregulation. After deregulation, despite the importance of cancellation forecasts to airlines revenue streams, methodological updates of cancellation models have been scarce. As a results, the current state of practice for cancellation models uses the same set of assumptions defined by the seminal work of Thompson (1961).

Another important finding of the literature review is that incorporating results of cancellation models in a way in which exact solutions for the general yield management practice are computationally feasible remains a challenging task (Talluri and Van Ryzin 2004a; Philips 2005). In this context, the majority of yield management solutions which aim to simultaneously solve overbooking and capacity allocation problems use heuristics. For exact solutions, experimental results of Subramanian, Stidham et al. (1999) point out

[22] AGIFORS – "the Airline Group of the International Federation of Operational Research Societies is a professional society dedicated to the advancement and application of Operation Research within the airline industry"(http://www.agifors.org/index.jsp , retrieved September 10th 2007)

the revenue impact of using class-dependent cancellation rates. Their findings mention that "close to optimal" approximation of class-dependent cancellation rates are bound to generate close-to-optimal solutions.

In contrast to the state of practice, findings of Chatterjee (2001) and Westerhof (1997) shed new light to the area of cancellation models. Their empirical evidence on the violation of the memoryless property of cancellation probabilities and the influence of group effects on cancelation probabilities are the starting point of current dissertation. As such, current dissertation aims to analyze combined effects of days from departure and days from issue on the intensity of the cancellation process. The revenue impact of this forecasting exercise will be quantified using a single leg capacity control simulation.

In order to analyze time-to-event properties of the cancellation process from a financial perspective current dissertation uses ticketing data. The following chapter describes the data selection process and the main characteristics of the Airline Reporting Corporation (ARC) dataset.

Chapter 4: DATA SOURCES

The data sample used to analyze the intensity of the cancellation process is unique from other studies in that it captures a mix of different markets and carriers from a ticketing perspective. Supported by Boeing Commercial Aircraft (BCA), the data collection efforts focused on choosing a disaggregate data source that fit the competitive market research objective and minimized data collection costs. The following sections motivate the use of Airline Reporting Corporation (ARC) data, contrast the available ticketing and booking data sources, and present main characteristics of the ARC data sample.

4.1. Ticketing versus Booking Data Sources

While both tickets and bookings can result in cancellations, the way in which cancellations events are recorded and the way in which data reflects real-world inventory operations are different. As a subset of booking data, ticketing data is a consolidated output of a ticketing clearing house which captures financial triggered events (*i.e.*, purchases, refunds and exchanges). In contrast, booking data captures events triggered by airline reservation systems (*i.e.*, bookings and cancellations).

In order to select a data source that matches the scope of current research, several options were explored: (1) the Market Information Data Transfer (MIDT) data generated by global distribution systems (GDS), (2) the ticketing clearing houses data generated by Airline Reporting Corporation (ARC), and (3) the Origin-Destination Data Bank (DB 1A or 1B) generated by the United States Department of Transportation (U.S.- DOT). Available data sources were compared across five dimensions: data unit, granularity, masked information, revenue stream resolution, and presence of control policies (see Table 4-1).

Table 4-1: Characteristics of the Available Data Sources

Data Set	Data unit	Granularity	Masked Information	Revenue Stream Resolution	Control Policies
MIDT	Bookings	Disaggregated	Fare; Carrier; Passenger Identity	NA	NA
ARC	Tickets	Disaggregated	Carrier; Passenger Identity	Unearned Revenue or Air Traffic Liability (ATL)	NA
DB 1A/1B	10 % of Flight Used Coupons	Aggregated	None	10% of Earned Revenue	NA

With respect to the data unit, the MIDT dataset is the most complete[23] dataset capturing customers' requests through different channels at a reservation level. In contrast, the DB 1A/1B dataset is the least complete, capturing only 10% of the flight coupons. Although MIDT and ARC datasets are disaggregated, the available information and the resolution of the revenue stream are richer in the DB1A/1B dataset. Finally, information about the type and the frequency of inventory and overbooking controls is missing in all datasets.

Since this research focuses on analyzing airline passenger behavior from a financial perspective the ARC dataset is desirable to use. Still, with carrier information masked and sales reporting procedures subject to settlement systems agreements, the connection between ticketing data and revenue management algorithms and heuristics remains to be explored. Specifically, the equivalence between cancelled bookings and refunded and exchanged tickets needs to be defined.

As shown in Figure 4-1, the set of collectively exhaustive and mutually exclusive booking states is defined by churn bookings, cancellations, no-shows, standbys, and

[23] Here we refer to the MIDT ability to capture booking events trough different distribution channels. However current evidence from Coldren, G. M., F. S. Koppelman, et al. (2003) points out that internet sales through GDS are declining

shows. Depending on the type of ticketing event, ticketed bookings are associated with a one of the following booking states: cancellation, no-show, standby, and show.

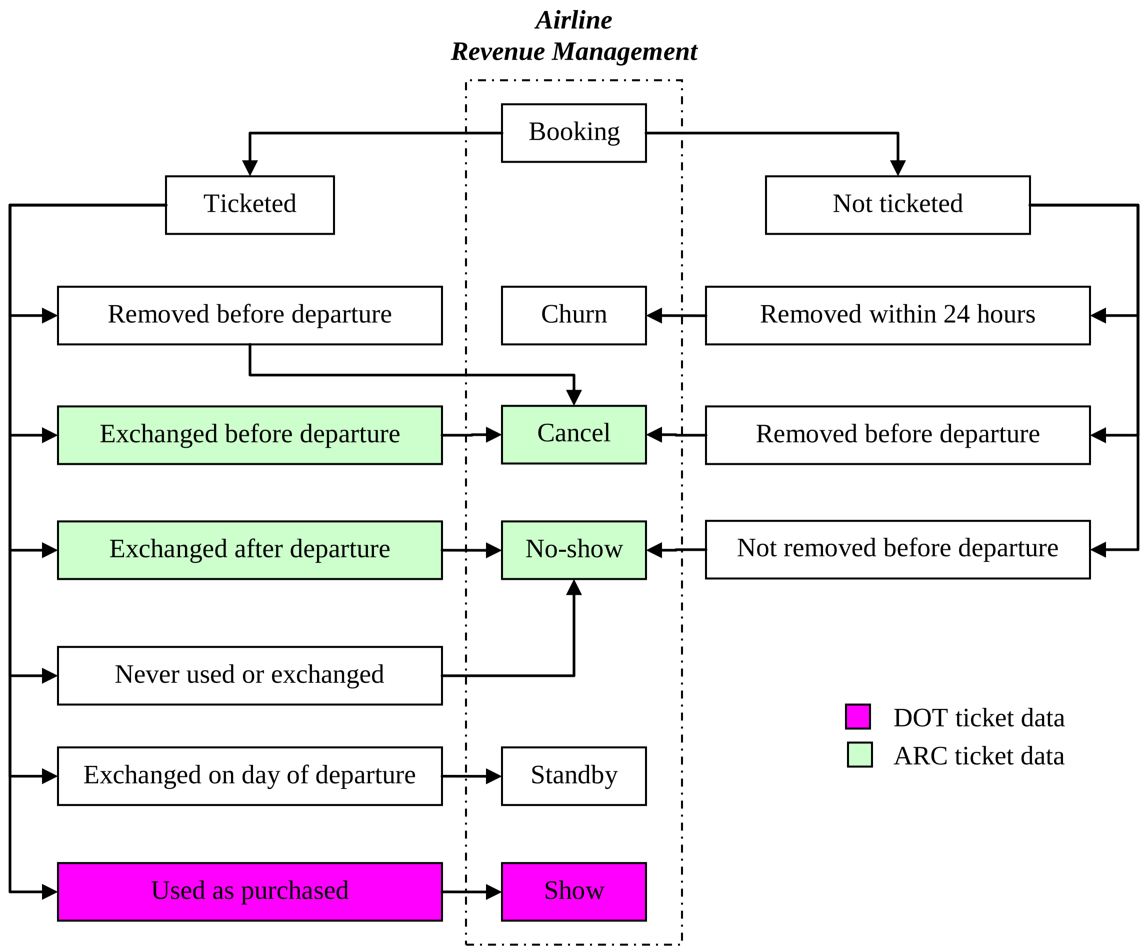

Figure 4-1**:** Relationships among Bookings, Tickets and Cancellations, No-Shows

The four booking states are equivalent to ARC ticketing events (refunds, exchanges and voids) and are triggered by financial transactions. For example, when a passenger informs the airline prior to departure that she/he does not intend to take the ticketed flight the original booking associated with the ticket is cancelled from the RM system and a new booking (and ticket transaction) is created for the new flight(s) purchases. These transactions appear in the ARC dataset.

There are some cancelled ticketed transactions that will not appear in the ARC dataset. For example, some airlines use automated data processes that cancel the inbound segments of an itinerary if the passenger no-shows on the outbound segments. In this case, the outbound segments that were never used or exchanged prior to departure become no-shows and the carrier automatically cancels in the inbound segments (without generating an automatic refund / exchange transaction).

Similar to cancellations, there are two ways in which a ticketed booking can become a no-show and only one of these cases appears in the ARC data. No-shows that occur due to exchanges or refunds requested after the flight departure are captured in ARC ticketing data. No-shows that occur when an individual purchases a ticket yet never uses it or purchases a ticket and requests a refund after the outbound departure date are not captured in the ARC data.

In contrast to no-shows and cancellations, a show occurs when a ticket is used exactly as purchased. This "snapshot" of tickets is what is captured in the lifted tickets collected in the DB 1A dataset. Finally, it is important to note that changes to tickets that occur on day of departure for the flight are not captured in the ARC data, but rather are part of an individual airline's check-in processing.

4.2. The Airline Reporting Corporation (ARC) Ticketing Data

It is important to reiterate that the ticketing dataset used for this study is distinct from the industry default source, *i.e.*, the Origin and Destination Data Bank 1A or Data Bank 1B (commonly referred to as DB 1A).

Collected from passengers as they board aircraft operated by U.S. airlines[24] the DB 1A dataset is based on a 10 percent sample of flown tickets. Supported by the U.S. DOT, the dataset provides demand information on the number of passengers transported between origin-destination pairs, itinerary information (marketing carrier, operating carrier, class of service, etc.), and price information (quarterly fare charged by each airline for an origin-destination pair that is averaged across all classes of service). While raw DB datasets are commonly used in academic publications (after going through some cleaning to remove frequent flyer fares, travel by airline employees and crew, etc.), airlines generally purchase Superset[25] data from Data Base Products.

In contrast to DB 1A, the ARC dataset captures ticketing transactions such as purchases, refunds, and exchanges across multiple airlines and multiple markets. To support research objectives while protecting airline confidentiality each individual ticket used in current analysis had the airline codes replaced by a randomly assigned number and the flight information (including flight numbers, departure and arrival times, number of stops, etc.) suppressed. A complete description of the data fields present in the ARC data set is presented in Appendix A.

From a modeling perspective, it is generally believed that cancellation rates differ for business and leisure passengers. For example, business passengers who are more time-sensitive and require more travel flexibility may be more likely to modify their

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[24] "The raw materials for the Origin-Destination survey are provided by all U.S. certificated route air carriers, except for a) helicopter carriers, b) intra-Alaska carriers, and c) domestic carriers who have been granted waivers because they operate only small aircraft with 60 or fewer seats." Data Base Products (2006). "The origin-destination survey of airline passenger traffic." Retrieved April 30, 2006, from http://www.airlinedata.com/Documents/O&DSURV.htm.

[25] Superset is a cleaned version of the DB data that is cross-validated against other data-sources to provide a more accurate estimate of the market size. See the Bureau of Transportation Statistics website at www.bts.gov or the Data Base Products, Inc. website at www.airlinedata.com for additional information

itineraries than leisure passengers, leading to higher cancellation and no show rates. While airlines do not explicitly collect information about trip purpose, trip purpose can be inferred from several other booking, non-directional itinerary, and directional itinerary variables. An itinerary is defined as a flight or sequence of flights that connects an origin and destination. Non-directional itinerary information does not distinguish whether passengers on a flight from MIA-SEA are traveling outbound from MIA to SEA or inbound from SEA to MIA. While non-directional information is predominately used in airline's RM systems, directional itinerary information provides a much richer set of variables from which trip purpose can be inferred. For example, business passengers are more likely to depart early in the week, stay a few nights, and return home later in the week (and thus not stay over a Saturday night). In contrast, leisure passengers are more likely to depart later in the week, stay more nights than a business passenger.

The ARC dataset contains directional[26] one-way and round-trip tickets with the outbound departure date on 2004. To capture a mix of business and leisure markets and a mix of round trip and one ways a total of eight directional markets are included in the analysis. Each market is served by at least three airlines and contains non-stop and connecting itineraries. The markets include travel in origin destination pairs involving Miami, Seattle, or Boston (specifically, MIA-SEA, SEA-MIA, MIA-BOS, BOS-MIA, BOS-SEA, SEA-BOS) in addition to travel between Chicago O'Hare airport and Honolulu (ORD-HNL, HNL-ORD).

Overall, 1.3% of the tickets are refunded and 1.2% exchanged, but there are large differences across markets (see, Table 4-2

[26] A "simple" ORD-HNL one-way itinerary is one in which the trip starts in ORD and ends in HNL. The passenger is embarks at ORD (*i.e.*, there are no flight segments before ORD) and disembarks at HNL (*i.e.*, there are no flight segments after HNL). Similar logic applies to round-trip itineraries.

). While carrier confidentially considerations restrict the amount of flight-level information available for analysis, the ARC sample data is unique in its ability to capture information about the time until exchange and refund events across multiple markets and multiple carriers.

Table 4-2: Refund and Exchanges by Market and Trip Type

Market	*# tickets*	*# (%) Refunded*		*# (%) Exchanged*		*# (%) One Ways*		*# (%) Round Trips*	
MIA-SEA	8,599	623	(7.2%)	84	(1.0%)	4,095	(48%)	4,504	(52%)
SEA-MIA	18,059	210	(1.2%)	198	(1.1%)	3,433	(19%)	14,626	(81%)
BOS-MIA	84,752	858	(1.0%)	1,248	(1.5%)	9,013	(11%)	75,739	(89%)
MIA-BOS	23,800	106	(0.4%)	318	(1.3%)	9,778	(41%)	14,022	(59%)
BOS-SEA	35,204	374	(1.1%)	423	(1.2%)	6,337	(18%)	28,867	(82%)
SEA-BOS	34,564	288	(0.8%)	442	(1.3%)	6,178	(18%)	28,386	(82%)
HNL-ORD	5,261	62	(1.2%)	51	(1.0%)	1,715	(33%)	3,546	(67%)
ORD-HNL	24,131	416	(1.7%)	138	(0.6%)	1,664	(7%)	22,467	(93%)
TOTAL	234,370	2,937	(1.3%)	2,902	(1.2%)	42,213	(18%)	192,157	(82%)

Besides market and carrier information, the ARC dataset includes several other ticketing characteristics: the issue date (or date the ticket was purchased), the outbound and inbound departure dates, outbound and inbound ticketing class (*i.e.*, first letter of the fare basis code), ticketing cabin code (*i.e.*, first, business, coach, other/unknown), net fare (*i.e.*, fare that does not include taxes and fees), and total tax and fees. Also, tickets that are refunded or exchanged contain the refund or exchange date and the exchange fee/fare difference from the original ticket. Furthermore, indicator variables which show the reason for which that ticket was exchanged are also populated. Specifically, indicators are used to know whether the customer requested (1) a new outbound and/or inbound departure date, (2) a new outbound and/or inbound ticketing class and cabin code, and/or (3) a new outbound and/or inbound itinerary. Characteristics related to trip purpose (*i.e.*, Saturday night indicator), week seasonality (outbound departure day of the week) and carrier were inferred from the available data.

In addition, using the outbound and the inbound departure dates several segmentation variables were derived. Table 4-3 presents the percentage of refunds and exchanges across different advance purchase periods. For tickets purchased 8 to 360 days from the outbound departure date, exchanges exhibit a "tub" shape, characterized by a higher propensity of tickets purchase well in advance from departure or in the 2 to 3 weeks from departure to be exchanged.

Table 4-3: Refund and Exchanges by Advance Purchase

Advance Purchase	*Exchanges*		*Refunds*		*Exchange & Refunds*		Total Tickets
0-3	33	0.151%	267	1.223%	300	1.375%	21,825
4-7	245	1.155%	483	2.278%	728	3.433%	21,205
8-14	478	1.731%	430	1.558%	908	3.289%	27,607
15-21	426	1.593%	312	1.167%	738	2.760%	26,738
22-30	410	1.323%	370	1.194%	780	2.517%	30,988
31-40	333	1.175%	445	1.570%	778	2.745%	28,344
41-50	236	1.077%	259	1.182%	495	2.260%	21,904
51-90	421	1.397%	197	0.654%	618	2.051%	30,126
91-180	259	1.559%	118	0.710%	377	2.269%	16,618
181+	61	1.921%	56	1.763%	117	3.684%	3,176

Also, is worth noticing that exchanges drop dramatically one week from departure. In contrast, refunds tend to increase as the advance purchase decreases, that is, tickets purchased closer to the outbound departure date are more likely to be refunded. However, at 31-40 days from departure, there is a slight increase in the percentage of refunds which may be attributed to consolidator bookings (such as air travel associated with cruise lines that are present in the Miami and Seattle markets). Also, similar to exchanges, the percent of refunds drops very close to departure, or 0-3 days from the outbound departure date.

Table 4-4 presents the refund and exchanges rates across a popular proxy for business-leisure segmentation, *i.e.*, the Saturday night stay. As expected, round trip tickets with a Saturday stay (that tend to be associated with leisure travel) are less likely to be exchanged or refunded than round trip tickets without a Saturday stay (that tend to be associated with business travel).

Table 4-4: Refund and Exchanges by Saturday Night Stays (Round Trip Tickets)

Saturday Stay	*Exchanges*		*Refunds*		*Exchange & Refunds*		Total RT Tickets
Saturday Stay	1,401	1.092%	1,263	0.984%	2,664	2.076%	128,333
No Saturday Stay	1,476	2.313%	1,147	1.797%	2,623	4.110%	63,824

Differences in exchange and refund rates between business and leisure travelers are also seen in Table 4-5 - the effect of outbound departure dates. Exchanges are more likely to occur on Sunday, Monday, and Tuesday outbound departures and Wednesday, Thursday, and Friday inbound returns. Refunds exhibit a similar pattern, but also show a relative high rate on Saturday outbound departures.

Table 4-5: Refund and Exchanges by Outbound Day of the Week

Day of Week	*Exchanges*		*Refunds*		*Exchange & Refunds*		Total RT Tickets
Sunday	390	1.219%	564	1.763%	954	2.982%	31,989
Monday	515	1.684%	494	1.616%	1,009	3.300%	30,575
Tuesday	461	1.790%	337	1.308%	798	3.098%	25,759
Wednesday	477	1.548%	342	1.110%	819	2.659%	30,806
Thursday	416	1.066%	316	0.810%	732	1.876%	39,017
Friday	375	0.922%	390	0.959%	765	1.882%	40,653
Saturday	268	0.753%	494	1.389%	762	2.142%	35,571

Finally, Table 4-6 shows the exchange and refund rates by month of the outbound departure date and refunds and exchanges. No clear pattern can be detected, suggesting the effects of seasonality may be limited. In addition to the variables described above, tickets that are refunded or exchanged also contain the date the refund or exchange was processed. In addition, when one ticket is exchanged for another ticket, information on the exchange fee and fare difference from the original ticket is available. Indicator variables are also populated to show the reason for the exchanged ticket. Specifically, indicators are used to know whether the customer requested (1) a new outbound and/or inbound departure date, (2) a new outbound and/or inbound ticketing class and cabin code, and/or (3) a new outbound and/or inbound itinerary.

Table 4-6: Refund and Exchanges by Month of Departure Date

Departure Month	*Exchanges*		*Refunds*		*Exchange & Refunds*		Total RT Tickets
January	245	1.412%	266	1.53%	511	2.94%	17,357
February	233	1.041%	306	1.37%	539	2.41%	22,384
March	237	0.983%	277	1.15%	514	2.13%	24,108
April	226	0.966%	264	1.13%	490	2.09%	23,402
May	244	1.262%	199	1.03%	443	2.29%	19,332
June	251	1.325%	239	1.26%	490	2.59%	18,946
July	204	1.136%	244	1.36%	448	2.49%	17,961
August	186	1.029%	202	1.12%	388	2.15%	18,071
September	273	1.693%	325	2.02%	598	3.71%	16,124
October	279	1.532%	224	1.23%	503	2.76%	18,209
November	293	1.604%	167	0.91%	460	2.52%	18,267
December	231	1.143%	224	1.11%	455	2.25%	20,209

To summarize, unlike DB 1A ticketing data or booking data from a single airline, ARC ticketing data provides an opportunity to develop no-show and cancellation models for multiple airlines and/or markets. Most important, the no-show and cancellation rates derived from ARC ticketing data *directly tie to the revenue generation stream of an airline,* which is one of the most important metrics to an airline considering aircraft purchases.

Chapter 5: RESEARCH METHOLOGY

The main focus of this dissertation is to explore the use of discrete oriented methods for airline "cancellation" models. Based on the occurrence of refund, exchange events in a ticketing dataset (*i.e.*, the ARC dataset), conditional probabilities (hazard probabilities) of purchased tickets experiencing the event of interest are predicted. Survival analysis methods are used to explore the pattern of cancellation probabilities over time and to determine the extent in which the observed heterogeneity of tickets (*i.e.*, predictors) changes that pattern.

This section contains several parts. First the research objectives are stated. Second, an overview of the key concepts of time-to-event modeling is provided and the use of a Discrete Time Proportional Odds (DTPO) model to forecast airline passenger cancellations is motivated. Third, a simulation exercise to assess the impact of time-to-event forecasts on revenue streams is presented.

5.1. Research Objectives

This research contributes to the literature in three distinct ways. First, with respect to air travel behavior, it is the first study of airline passengers' cancellation behavior based on survival methods. In comparison to cancellation models reported in the literature or used in practice, the proposed framework is more "customer-focused" in the sense that it captures the underlying behavior of passengers. In that perspective, the impact of time from purchase, time until departure, and directional itinerary and booking covariates on the intensity of cancellation process is explored.

Second, with respect to the data used, this research introduces a different ticketing data source that the industry' default (*i.e.*, the origin and destination Data Bank 1A. In contrast to the DB 1A dataset, the ARC dataset captures cash triggered transactions (refunds, exchanges and voids) across multiple carriers and multiple markets, providing unique opportunities to analyze the ticketing process from a financial perspective.

Finally, with respect to revenue management methodological advances, the current research addresses the validity of some of the common assumptions associated with previous cancellation research and tests the applicability of present cancellation models in the context of the general seat inventory control problem.

5.2. Airline Passenger Cancellation Behavior and Time-to-Event Analysis

In the context of current state of practice for cancellation models, the following sections motivate and describe the time-to-event procedures of current research. Section 5.2.1 presents the general concepts of time to event analysis. Section 5.2.2 presents time to event model selection procedures. Finally, Section 5.2.3 describes the estimation of a Discrete Time Proportional Odds model.

5.2.1. General Taxonomy of Survival Analysis Models

Survival models are designed to analyze data for which the response variable is defined as a time to an event(s). In contrast to classical linear regression methods, survival models exhibit two notable features: (1) the presence of censored data, and (2) the possibility of time-varying covariates (McCullagh and Nedler 1989). Both aspects are governed by a "time at risk" mechanism in which the dynamics of conditional probabilities of an event happening (*i.e.*, the transition intensity) are assessed as a function of the elapsed time since the entry time.[27]

Statistical methods for survival data were developed to support epidemiological applications (that capture the time-to-occurrence of an event given exposure to an infection) or clinical applications (that capture the time-to-occurrence of an event given exposure to treatment). The fundamental difference between the two categories of studies consists in the way survival time is considered – either in retrospective or prospective (Kim and Lagakos 1990). In retrospective studies, investigators analyze the disease incidence for exposed individuals “in hindsight” based only the prevalence of disease at the time the data is collected (Shiboski 1998). In contrast, in prospective studies investigators use a “forward looking” approach to analyze the evolution of disease for individuals exposed to various treatments (Hosmer and Lemeshow 1999).

Although, survival analysis concepts were first tested and validated by the medical field their applicability to demography, econometrics, travel demand, and other areas was immediate. Today, a multitude of methodological “add-ons” are testimonial to the degree of generalization that survival analysis concepts have reached and comprehensive reviews are provided by several authors (Kiefer 1988; Jain and Vilcassim 1991; Hensher and Mannering 1994; Bhat 2000; Wu 2003).

Since survival “methods are so similar in their underlying philosophy that they usually give similar results” (Allison 1995) the choice of “the right” survival model depends on several substantive assumptions regarding the population at risk, the

[27] In the case when entry time is the same as the time when the subject becomes at risk. This might not always be the case (*e.g.* delayed entry).

beginning and end of an observation, the censoring mechanism, the distributional assumptions about the time-to-event process, and the choice of the dependent variable.

The population at risk is defined as independent "subjects" under observation during parts or the entire period of a survival study. The beginning of an observation is identified by the time at which the subject becomes at risk of "dying." The end of an observation is identified by the time at which the event is observed (non-censored) or by the time at which the follow-up process ends (censored). With respect to the distributional assumptions, survival models are categorized as continuous time semi-parametric, continuous time parametric, discrete time, and non-parametric. Finally, the choice of the dependent variable, *i.e.*, survival time $S(t)$ vs. hazard rate $h(t)$, influences the way in which covariates effects[28] are interpreted. For accelerated failure time (AFT) models a base survival time is accelerated. In contrast, for proportional hazard (PH) models a base hazard rate is multiplied.

5.2.2. Model Selection

In the context of survival analysis, ARC ticketed transactions represent n independent "subjects at risk" of a cancellation[29] event. The time until the occurrence of a cancellation event or the time until departure (t) is a continuous or discrete non-negative random variable which represents the "observed life" of a ticket. If $f(t)$ is the probability distribution function or probability mass function associated with the time-to-event process, the intensity of the cancellation process can be described by the survival time

[28] For AFT models coefficients of covariates represent changes in survival time due to a unit change in a given covariate while for PH models coefficients represents changes in the hazard rates due to a unit change in a given covariate

[29] Identified by refund of exchange events.

S(t) or the hazard rate *h(t)* (see Equation 5-1 for the continuous time case and Equation 5-2 for the discrete time case [30]).

$$S(t) = \Pr\{T > t\} = 1 - F(t) = \int_t^{\infty} f(x)dx$$
$$h(t) = \frac{f(t)}{S(t)} \tag{5-1}$$

$$S(t_j) = S_j = \Pr\{T \geq t_j\} = \sum_{k=j}^{\infty} f_j$$
$$h(t_j) = h_j = \Pr\{T = t_j \mid T \geq t_j\} = \frac{f_j}{S_j} \tag{5-2}$$

In selecting the most appropriate time-to-event specification to model the intensity of the cancellation process, two topics are particularly relevant. The first topic relates to how to "appropriately" specify models when multiple time dimensions are present. This problem is not new and is frequently encountered in life course demographic studies based on cohort datasets. Specifically, an underlying identification problem exists because given knowledge of the respondents' age and duration in the study, their cohort (or entry in the study) is uniquely determined (Wu 2003). In the context of cancellation models, this issue is relevant when testing for the validity of the memoryless property (*i.e.*, how to simultaneous consider both the days from departure and days from issue). The second topic relates to using the most appropriate assumptions to capture the specific characteristics of the time-to-event data. Two categories of assumptions characterize the time-to-event models: (1) distributional assumptions about

[30] Note that if in the case of the continuous time the hazard rate represents an **instantaneous rate of occurrence**, in the case of discrete time is a **conditional probability**.

the dependent variable, and (2) assumptions about the influence of the vector of covariates on the time-to-event process.

Table 5-1 presents the main categories of models considered for model selection. If time from issue effect is ignored, cancellation probabilities of bookings on hand can be determined by estimating a series of binary logits. Although similar in spirit with current state of practice[31], this approach has two caveats. First, it does not capture the transitional properties of the cancellation process. In a very dynamic environment such as airline industry, one might be interested not only to know the proportion of passengers to cancel by departure date but also the daily rate of this phenomenon. Second it requires the maintenance and estimation of multiple logit models, which increases the computational burden of yield management system.

Table 5-1: Time-to-event Models and Distributional Assumptions

Time-to-Event Models	Distributional Assumptions
1. No Time Scale	Binary Logit
2. Accelerated Failure Time (AFT) Class	Exponential , Weibull, Log-logistic, Log-normal
3. Proportional Hazard (PH) Class	Exponential , Weibull , Gompertz
4. Semi-Parametric Class	COX Proportional Hazards, Piece-wise exponential
5.Time-scale as a Covariate	Binary Logit, Complementary Log-log

As an alternative to a series of binary logit models, time-to-event models focus on analyzing cancellation process as a function of time from issue. To determine the most appropriate model, current research contrasts several time-to-event formulations (rows two to five from Table 5-1). The first two categories of models, *i.e.*, the accelerated failure time and the proportional hazard identify the variables of interest used in time-to-event analysis: survival time and hazard rate.

[31] Cancellation probabilities are equivalent with cancellation rates presented in Chapter 2

Accelerated failure time models are a special case of generalized linear models which focus on analyzing time-to-event processes from a survival time perspective. If t is a random variable to describe the observed/ unobserved survival time or time until failure, than, the effect of exponential values of covariates on t is multiplicative, *i.e.*, base survival time is either accelerated or decelerated (see Equation 5-3). In the context of model selection procedures, several distributions were used to describe the random disturbance term: standard extreme value (exponential model), extreme value two parameters (Weibull model), logistic (log-logistic model), and normal (log-normal model).

$$\log(t) = X'\beta + \sigma \cdot \epsilon \tag{5-3}$$

In contrast to AFT models, proportional hazard models focus on analyzing time-to-event processes from a conditional intensity (hazard) perspective. When compared to a baseline hazard, the effects of covariates are multiplicative. Also, the heterogeneity across observation is considered to be fully described by hazard variation, *i.e.*, two observations with identical values of covariates have identical values of hazards (see Equation 5-4). In the context of PH models, several shapes were used to describe the base-line hazard: constant (exponential model), linear (Gompertz model), and linear in the logarithm of time (Weibull model).

$$\log[\mathrm{h}(t)] = \log\,[\mathrm{h}_0(\mathrm{t})] + X'\beta \tag{5-4}$$

The third category of time-to-event models considered for model selection is the semi-parametric class. As one of the most popular time-to-event models, the COX proportional hazard model estimates the relative risk of an event happening. In this context, the effect of baseline hazard is clearly separated from the effects of covariates.

The COX model can be interpreted as a proportional hazard model or an additive model in the log of hazards (see Equation 5-5). The piece-wise exponential model constraints the proportionality assumption of COX model across segments of time. Instead, the unconstrained base-line hazard of the COX model is replaced by a succession of piece-wise constant baseline hazards (see Equation 5-6)

$$\log[h(t)] = h_o(t) + X'\beta \tag{5-5}$$

$$h_0(t) = \gamma_i \quad where\ \ i\ in\ [a_{i-1}, a_i) \tag{5-6}$$

The models from the last category of time-to-events models represent "equivalent" formulation of COX proportional hazard model which use maximum likelihood estimation instead of GLM methods. In the context of events happening at discrete point in times or continuously, hazard estimates of discrete time proportional odds (DTPO) model and the complementary log-log model (CLL) represent good approximation of the proportional hazards scenario.

For this research, two arguments favor the use of the DTPO model as the appropriate formulation to estimate the pattern of tickets' cancellation probabilities. The first refers to the computational efficiency of the maximum likelihood (ML) estimators when compared to the partial likelihood (Kaplan and Meier 1958) estimators. Indeed, since the ARC sample dataset is a "consolidated" dataset, with tickets aggregated from eight different markets, the presence of a large number of ties is inevitable, a fact that eliminates the alternative of an exact Cox model estimation.

The second refers to an on-going debate in the revenue management field as to which is the most appropriate model to describe how cancellation probabilities evolve over time. Although several authors indicate that the value of cancellation probability is constant over time (Littlewood 1972) and independent of the time of booking (Talluri and Van Ryzin 2004a), empirical evidence suggests otherwise (Westerhof 1997; Chatterjee 2001).

To conclude, the DTPO model offers the flexibility of testing different scenarios with minor adjustments. In view of these advantages, the next section describes the DTPO model as an alternative way to estimate cancellation probabilities for the sample of ARC airline tickets.

5.2.3. The Discrete Time Proportional Odds (DTPO) Model and ARC data

As mentioned before, the study of cancellation determinants in the framework of ticketing data is a new research area and comes to give further insights on findings of Garrow and Koppelman (2004a; 2004b) in the field of no-shows and standby behavior. The relevance of such a study is motivated by challenges that customers booking behavior pose on present airline carrier's financial stability.

The DTPO model extends previous research on the distribution of cancellation rates/proportions[32] in four aspects. First, it relaxes the general assumption of population homogeneity and tests the influence of observed heterogeneity on cancellation rates/proportions by considering different segmentations/covariates (Saturday night stay, outbound departure day of week, market, carrier, group size, pro-rated fare). Second, it assumes that heterogeneity across tickets is fully captured by these covariates and its effect is distinct from that of time (changes in covariates values produce only vertical shifts and no distortions in a "baseline" cancellation rate line, *i.e.*, the proportional hazard

[32] Chatterjee (2001) defines a cancellation rate at time t as the proportion of those booked at t which cancel by t-1 and a cancellation proportion at time t as the proportion of those booked at t which cancel by departure day. In contrast, Thomson (1961) and Taluri and Van Ryzin (2004a) define the cancellation rate as a the proportion of those booked at t which cancel by departure day.

assumption). Third, by construction, the DPTO model accommodates time-varying covariates, thus permitting the presence of multiple time scales (*i.e.*, days from issue and days from departure). Finally, since the time-scale is discrete, the DTPO model has sufficient flexibility to test different distributional shapes for the baseline cancellation rate.

It is important to note that compared to the typical time-to-event datasets, the ARC sample ticketing data has three unique characteristics. The first characteristic is that the tickets "lifetimes" are completely determined, and end either in a cancellation (exchange/refund date) or in certain non-cancellation (outbound departure date). As a result, a unique set of possible life-times is known for each ticket; that is, the set of possible lifetimes for a ticket is bounded between one and the difference between its departure and issue dates. While infrequently encountered in the context of survival analysis, this particularity of the data proves to be extremely useful in exploring whether the memoryless property of cancellation rates holds.

The second characteristic is that the assumption of independence between observations is undoubtedly violated by the presence of groups. Therefore, the ARC dataset was transformed from an individual ticket level database to a group level database. More specific, observations determined to have the same values on the entire set of covariates with the same scrambled passenger name record (PNR)[33] were eliminated and a variable indicating the group size added to the set of covariates. Also, taking into account that the majority of tickets are booked in the 0 to 90 days from departure (DFD) time interval (95% of total number of tickets) and cancellation events

[33] To ensure carrier and passenger confidentiality, ARC provided "scrambled" PNR information and ensured that these records were unique within a specific market. The PNR records provide information on how many passengers are traveling together on the same reservation.

for the rest of tickets are relatively scarce (5% out of total number of events), the ARC dataset was reduced to tickets booked 90 days from departure or earlier.

The third characteristic of ARC ticketing data that influences the application of survival methodology is that only refund and exchange events occurring prior to the outbound departure date are considered[34].

Figure 5-1 illustrates the distinction between the outbound and inbound portions of a simple round-trip itinerary. In this example, the passenger purchases a ticket to travel outbound – or to depart Boston for Seattle – on June 1. The same passenger plans to travel inbound – or to return from Seattle home to Boston – on June 4. The outbound itinerary includes a single flight leg while the inbound itinerary includes two flight legs to represent the connection at Chicago O'Hare airport (SEA-ORD and ORD-BOS).

[34] The methodology applied for outbound itineraries can be extended to inbound itineraries, albeit the "behavioral analysis" becomes slightly more complicated, as many airlines automatically cancel inbound itineraries once they know that the passenger has "no-showed" for the outbound itinerary.

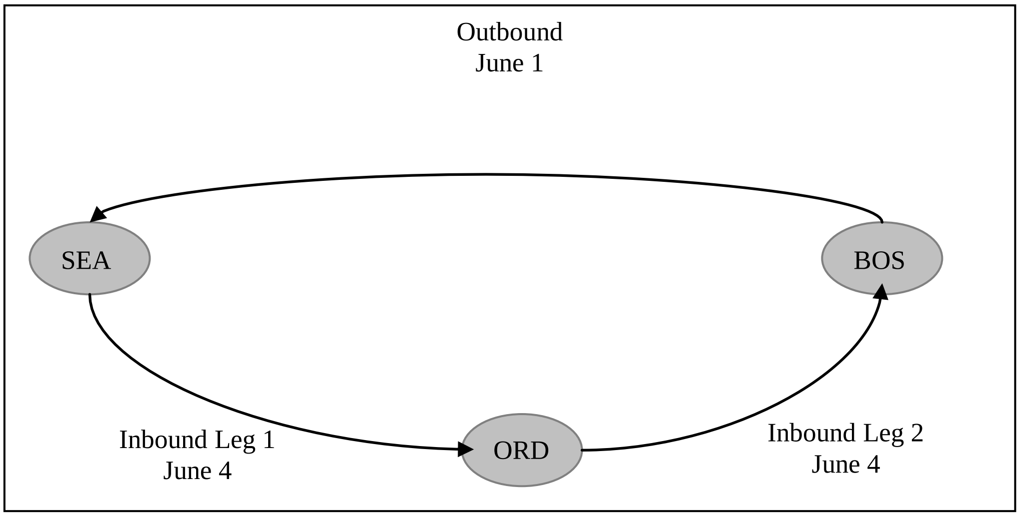

Figure 5-1: Outbound and Inbound Itineraries

For the purposes of this analysis, only refund and exchange events (assimilated into a single "cancellation event") that occur on the BOS-SEA outbound itinerary prior to June 1 are considered. This is because the primary interest of this study is to model cancellation behavior; refund and exchange events that occur to the BOS-SEA itinerary on or after June 1 appear as "no shows" within the current revenue management framework, since the airline does not know prior to the outbound flight departure that the passenger does not intend to travel. To give a general idea on the magnitude of the two problems Table 5-2 shows the distribution of total population refund and exchange events with respect to the outbound departure date.

Table 5-2: Percentage of Refund and Exchange Events

Type of event	*Percentage (out of total tickets)*	*Percentage (out of total events)*
Refund event **before or on** ODT	0.82%	27.08%
Exchange event **before or on** ODT	1.40%	46.28%
Exchange or refund event **on** ODT	0.19%	6.23%
Refund event **after** ODT	0.62%	20.61%
Exchange event **after** ODT	0.18%	6.03%

Given all of the above characteristics, the ARC time-to-event application can be viewed as a *ticketing cancellation model on the outbound legs of simple round-trip airline itineraries for groups for a ticketing horizon of 90 days from departure.* After this data reduction process, the original ARC dataset of 234,370 tickets (1.3% Refunds; 1.2% Exchanges) was transformed to 151,401 unique groups (2.22% Cancellations).

Using the transformed ARC data, the DTPO model partitions the time-to-event of the $\boldsymbol{i^{th}}$ ticket $\mathbf{(T_i)}$ into a number of k disjoint time intervals $\mathbf{(t_0, t_1], (t_1, t_2],(t_2, t_3], \ldots,(t_{k-1}, t_k]}$. The bounds of the time intervals $\mathbf{(t_0,t_1,\ldots,t_k)}$ identify the days from issue (DFI) where $\boldsymbol{t_0}$ represents the issue date and $\boldsymbol{t_k}$ represents either the time of departure (non-cancelled tickets) or the time of ticket refund/exchange (cancelled tickets). In this context, the discrete hazard of a cancellation event for the $\boldsymbol{i^{th}}$ ticket in the $\boldsymbol{k^{th}}$ interval is defined as the conditional probability that ticket $\boldsymbol{i}$ will experience the cancellation event in the $\boldsymbol{k^{th}}$ interval given survival up to that point (Equation 5-6). Using conditional probability theory, it follows that the probability that a cancelled ticket will experience the event in the $\boldsymbol{k^{th}}$ interval is equal to the product between the non-event conditional probabilities of $\boldsymbol{1}$ to $\boldsymbol{k}$**-1** time intervals and the event conditional probability of $\boldsymbol{k}$ time interval (Equation 5-7). Similarly, the probability that a non-cancelled ticket will experience the cancellation after the $\boldsymbol{k^{th}}$ interval is equal with the product of non-event conditional probabilities of all $\boldsymbol{k}$ time intervals (Equation 5-8).

$$h_{ik} = P(T_i = k \mid T_i \geq k) \tag{5-6}$$

$$\begin{aligned} P(T_i = k) &= \mathrm{P}(T_i = k \mid T_i \geq k) \cdot \mathrm{P}(T_i \neq k-1 \mid T_i \geq k-1) \ldots \mathrm{P}(T_i \neq 1 \mid T_i \geq 1) \\ P(T_i = k) &= h_{ik} \cdot (1 - h_{i(k-1)}) \cdot (1 - h_{i(k-2)}) \ldots (1 - h_{i1}) \end{aligned} \tag{5-7}$$

$$\begin{aligned} P(T_i > k) &= \mathrm{P}(T_i \neq k \mid T_i \geq k) \cdot \mathrm{P}(T_i \neq k-1 \mid T_i \geq k-1) \ldots \mathrm{P}(T_i \neq 1 \mid T_i \geq 1) \\ P(T_i > k) &= (1-h_{ik}) \cdot (1-h_{i(k-1)}) \cdot (1-h_{i(k-2)}) \ldots (1-h_{i1}) \end{aligned} \tag{5-8}$$

As a result, the likelihood contribution for cancelled and non-cancelled tickets can be expressed using Equations 5-9 and 5-10 and further detailed as the product of all the individual likelihoods (Equation 5-11) in which $\boldsymbol{c_i}$ is an indicator variable equal to 0 for cancelled tickets and 1 for non-cancelled tickets (Cox 1972).

$$L_i = h_{ik} \cdot \prod_{j=1}^{k-1} (1-h_{ij}) \tag{5-9}$$

$$L_i = \prod_{j=1}^{k} (1-h_{ij}) \tag{5-10}$$

$$L = \prod_{i=1}^{n} \left[h_{ik} \cdot \prod_{j=1}^{k-1} (1-h_{ij}) \right]^{1-c_i} \cdot \left[\prod_{j=1}^{k} (1-h_{ij}) \right]^{c_i} \tag{5-11}$$

Since the exact time of tickets transition from the state of not-cancelled to cancelled can be captured using a binary variable y_{ij} equal with 1 if ticket is cancelled in the j^{th} day from issue and 0 otherwise, it follows that Equation 5-12 is an alternative form to express the log-likelihood function. Moreover, the likelihood function for the entire sample (Equation 5-13) is equivalent with the likelihood function of a binary logistic regression model for which y_{ij} are assumed to be a collection of independent variables and whose data structure is expanded[35] to represent an unbalanced panel dataset (*i.e.*, each ticket observation is replicated multiple times, one time for each day from issue of the ticket lifetime).

[35] The creation of the expanded dataset process has several steps: (1) duplicating the set of time-invariant covariates over the entire life-time of a ticket, (2) filling in the time-variant covariates (if present) and (3) creating the binary indicators of the cancellation status y_{ij}.

$$l = \log L = \sum_{i=1}^{n}\sum_{j=1}^{k} y_{ij} \cdot \log\left(\frac{h_{ij}}{1-h_{ij}}\right) + \sum_{i=1}^{n}\sum_{j=1}^{k} \log\ 1-h_{ij} \qquad (5\text{-}12)$$

$$L = \prod_{i=1}^{n}\prod_{j=1}^{k} h_{ij}^{\ y_{ij}} (1-h_{ij})^{(1-y_{ij})} \qquad (5\text{-}13)$$

The equivalence between the two likelihood formulations (Equations 5-11 and 5-13) defines the rationale behind the DTPO model, a model introduced by Cox (1972) and further detailed by several authors (Brown 1975; Thompson 1977). For a general set of covariates X_{i}, Equation 5-14 presents the general formulation of the DPTO model, while Equations 5-15 and 5-16 present the estimation of hazard and survival probabilities.

$$\log\left(\frac{h_{ij}}{1-h_{ij}}\right) = \Psi_{ij} + \beta_1 \cdot X_{ij1} + \beta_2 \cdot X_{ij2} + ... + \beta_l \cdot X_{ijl} \qquad (5\text{-}14)$$

Where $\boldsymbol{\Psi_{ij}}$ - baseline hazard function, ***j = 1,2,…,k*** time intervals, ***i* = 1,2,…,*n*** observations and ***l* =** number of covariates.

$$h_{ij} = [1 + \exp(-(\Psi_{ij} + \beta_1 \cdot X_{ij1} + \beta_2 \cdot X_{ij2} + ... + \beta_l \cdot X_{ijl}))]^{-1} \qquad (5\text{-}15)$$

$$S_{ij} = (1-h_{i1})(1-h_{i2})...(1-h_{ik}) = \prod_{j=1}^{k}(1-h_{ij}) \qquad (5\text{-}16)$$

Before addressing the choice of functional form for the baseline hazard and the choice of covariates included in current analysis, it is important to note that the DTPO model is constructed on two fundamental assumptions. First, a linear relation between the covariates and the logistic transformation of ticket cancellation hazard is assumed (*linearity assumption*). Second, the effect of covariates over the odds of cancellation is considered to be constant over time (*proportionality assumption*). In view of these assumptions, the DTPO model formulation can be conceptualized as the multiplicative effect of the covariates' log-linear function on a baseline odds function (Equation 5-17)**.**

Also, when the magnitude of conditional probabilities is small (as is the case with the ARC data), Equation 5-17 indicates that the DTPO model is a close approximation of the proportional hazard (PH) model[36].

$$\frac{h_{ij}}{1-h_{ij}} = \frac{h^{0}_{ij}}{1-h^{0}_{ij}} e^{\beta_1 \cdot X_{ij1} + \beta_2 \cdot X_{ij2} + \ldots + \beta_l \cdot X_{ijl}} \tag{5-17}$$

In the context of DTPO model the effects of days from issue (DFI), days from departure (DFD), itinerary characteristics, and fare on ticket cancellation rates are explored. The time *from issue* and *days from departure* covariates are used to test if the memoryless property of cancellation rates holds. To assess the most appropriate shape for the baseline hazard (*i.e.*, DFI) non-parametric estimators of survival probability (Kaplan-Meier), cumulative hazard (Nelson-Aalen) and hazard rate (Cox-Oaks) are used (Hosmer and Lemeshow 1999). Once decided on the best DTPO fit, the DFD covariate is added to indirectly[37] isolate the effect of time of ticketing. As shown in Figure 5-2 the simultaneous presence of the two covariates permits the reconstruction of cancellation rates for each DFD and different times of ticketing. Finally, divided on three main categories: (1) *group size,* (2) *outbound departure day of week, Saturday night Stay* and, (3) *carrier*, *market* and, *pro-rated fare* the covariates describing the observed heterogeneity are added.

[36] The odds of a cancellation event will be approximately equal to the conditional probability of cancellation (*i.e.*, $h_{ij} \approx h_{ij} / (1- h_{ij})$).
[37] Since from a RM perspective the focus is on determining cancellation rates for DFD given different times of ticketing we decided to use DFD instead of time of ticketing.

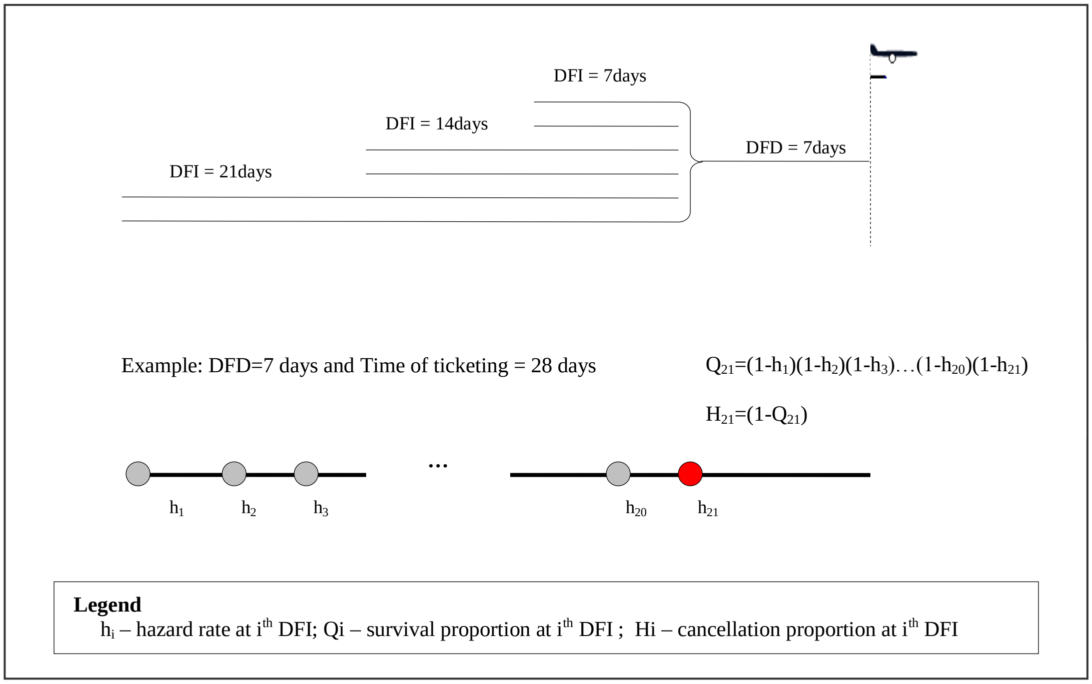

Figure 5-2: The Memoryless Property of Cancellation Rates- Conceptual Framework

To determine how ticket cancellations rates are influenced by proposed covariates several hypotheses are tested. First, with respect to the group and DFD effect, the results of Thomson (1961) indicate that cancellation proportions (defined as rates) decrease with group size and as the departure date approaches. Second, with respect to the business-leisure segmentation, one can hypothesize that variables associated with leisure passengers (Saturday night stay and Thursday, Friday and Saturday as outbound day of departure) result in a decrease of cancellation and no-show rates (*i.e.*, business passengers are more time-sensitive and experience frequent itinerary changes while leisure passengers are more price-sensitive and experience limited to none itinerary changes). Third, with respect to carrier information major carriers[38] (defined as those with market

[38] Due to confidentiality characteristics of the ARC dataset, the effects *of market*, *carrier* and *pro-rated fare* on cancellation rates are less clear.

shares greater than 10%) are expected to exhibit higher exchange and cancellation probabilities then smaller carriers (defined as those with market shares less than 10%). Finally, it is hypothesized that as fares increase, so too does the cancellation behavior. Intuitively, this is because business travelers, who tend to need more scheduling flexibility, are less price sensitive than leisure travelers.

5.3. Revenue Management Implementation of Time-to-Event Forecasts

To assess the value of estimation results to current revenue management (RM) state of practice, this research quantifies the impact of time-to-event overbooking controls on revenue streams. In this context, time-to-event forecasts of cancellations (based on new bookings) and classical forecasts of cancellations (based on bookings on hand) are successively applied to a simulation of a single resource capacity control and revenues are assessed. The difference between the two revenues streams is used to define the revenue opportunity of the time-to-event cancellation forecast.

Since exact solutions of the general yield management problem are rarely applied to real-world airline operations, the simulation exercise is designed as a collection of best industry practices (heuristics). Although not optimal, the current set of revenue estimation procedures adds to yield management state of practice by implementing time-to-event forecasts for cancellations.

In practice, a combined capacity allocation and overbooking heuristic consists of a set of sequential procedures applied iteratively over the entire length of booking horizon (Philips 2005):

- Forecast of the gross demand and cancellations
- Determine adjusted authorization levels using overbooking controls

- Determine capacity allocation controls (protection levels, booking limits or bid prices)
- Adjust capacity allocation control to match authorization levels
- Update the gross demand forecast and cancellation forecast over time and adjust overbooking limits and protection levels.

In the context of a dynamic capacity allocation and overbooking heuristic used in practice Table 5-3 presents the set of assumption used for current simulation. With simulation procedures covering forecasting, seat inventory, and overbooking areas, the set of assumptions was defined such it will closely match the current state of practice for a single resource capacity control.

With respect to forecasting assumptions Table 5-3 presents the distributional assumptions on the categories of demand and the types of forecasts used for each demand category. Fare class demand is normally distributed and results as a combination of independent Poisson processes. Constrained demand is uncensored with a Holt-Winters double exponential smoothing. Finally, future demand forecasts result from historical averages while future cancellation forecasts result from forecasts of the cancellation intensity (determined as a cancellation over bookings on hand ratio or as a hazard rate).

With respect to seat inventory control assumption, the Expected Marginal Seat Revenue (EMSRb) heuristic is used. Buy-ups and no-shows are not allowed. To determine overbooking limits, the Littewood algorithm (critical ratio) is used.

Table 5-3: The Set of Assumptions for Simulation

Forecasting Assumptions	
Model for demand	Normal distribution
Model for arrivals processes	Poisson distribution
Model for uncensoring	Holt-Winters DES
Forecasting Method for Demand	Aggregate (Historical Averages)
Forecasting Method for Cancellations (a)	Cancel/Bookings on Hand Ratio
Forecasting Method for Cancellations (b)	Hazard rates
Seat Inventory Control Assumptions	
Algorithm/ Heuristic	EMSRb (nested fares)
Update of booking limits	Dynamic
Scale	Single-leg Inventory Control
Buy-up behavior	No
Arrival of fares	Low-before-high(LBH) fare
Independece of demand for classes	Yes
Cancellations and no-shows	Only cancellations
Overbooking Control Assumptions	
Model for cancellation	Poisson
Overbooking algorithm	Littlewood rule (critical ratio)
Cost of overage	Highest Fare
Cost of underage	Lowest Fare

To motivate the use of current set of assumptions, the following sections present an overview of the main revenue management methodologies used in the simulation. Section 5.3.1 describes the heuristic used to allocate available capacity: Expected Marginal Seat Revenue (EMSR). Section 5.3.2 presents the risk-based overbooking algorithm used to determine overbooking controls. Finally, Section 5.3.2 presents the simulation used to assess the impact of time-to-event forecasts on airlines revenue streams.

5.3.1. Expected Marginal Seat Revenue (EMSR) Heuristics

Build on Littlewood's two-class model and refined by Belobaba (1989) the Expected Marginal Seat Revenue (EMSR) and its variants (EMSR-a, EMSR-b and EMSR-b with buy-up probabilities) represent one of the most popular heuristics used to determine seat-inventory controls (*i.e.*, protection levels) for static and dynamic *n*-class single-resource models.

Although not optimal, EMSR heuristics are popular among revenue management practitioners. The main idea of expected marginal analysis is to control the *n*-class inventory in such a way that prices of lower fare classes do not exceed expected marginal values of higher classes. For two classes with fares p_1 and p_2 (highest to lowest index), D_1 the demand for the highest class and x the remaining capacity, the allocation problem reduces to the classic newsvendor problem or Littlewood's rule (see Equation 5-18).

$$p_2 \geq p_1 \cdot P(D_1 \geq x) \tag{5-18}$$

As a first extension of the newsvendor problem, the EMSR-a heuristics determines protection levels by "applying Littlewood's rule to successive pairs of classes" (Talluri and Van Ryzin 2004a). For n-classes with fares $p_{i;\, i=n,n-1,\ldots,1}$ (lowest to high arrival order), D_i the demand for the class i, and x the remaining capacity, the EMSR-a formulation is described by the Equation 5-19:

$$p_i \geq p_{i-1} \cdot P(D_{i-1} \geq x)\,;\, i = n, n-1, \ldots, 2 \tag{5-19}$$

To determine protection levels, the EMSR-b heuristic adjusts the classical Littlewood rule to account for pooling effects. The fare of class i results as a weighted average ($\overline{p_\iota}$) of higher classes fares (see Equation 5-20). Also, the mean ($\overline{\mu_\iota}$) and standard deviation ($\overline{\sigma_\iota}$) of class i results as a sum of means and standard deviations of higher fare classes (see Equation 5-21).

$$\overline{\boldsymbol{p}_\iota} = \frac{\sum_{j=1}^{i} \mu_j \cdot p_j}{\sum_{j=1}^{i} \mu_j} \tag{5-20}$$

$$\mu_i = \sum_{j=1}^{i} \mu_j \;;\; \sigma_i = \sum_{j=1}^{i} \sigma_j \tag{5-21}$$

The EMSR-b heuristic can be adjusted to incorporate passenger no-show (Belobaba 1989) and buy-up behavior (Weatherford and Bodily 1992). In the presence of cancellations or no-shows, expected marginal revenues and fares are “deflated” with overbooking factors *OV* (see Equation 5-22). In the case of passenger diversion from lower to higher fares, the expected marginal revenue and fares are adjusted with the probability of sell-up *s* (see Equation 5-23).

$$p_i \cdot \frac{1}{OV_i} \geq p_{i-1} \cdot P(D_{i-1} \geq x) \cdot \frac{1}{OV_{i-1}} \tag{5-22}$$

$$p_i \geq p_{i-1} \cdot P(D_{i-1} \geq x) \cdot (1-s) + p_{i-1} \cdot s\,; i = n, n-1, \ldots, 2 \tag{5-23}$$

5.3.2. Risk-based Overbooking Algorithms

Since the objective of current simulation is to address the general yield management problem, overbooking algorithms have to be analyzed in the context of combined capacity control and overbooking algorithms. The following paragraphs describe the main characteristics of risk-based algorithms, contrast the use of exact solutions with the use of heuristics for the general yield problem, and discuss differences between current simulation and the state of practice.

With respect to the type of overbooking algorithm used several aspects are worth noticing. First, compared with service-based overbooking algorithms, risk based overbooking algorithms have the advantage of determining overbooking levels based on economic criteria. Second, the results of risk-based overbooking models, *i.e.*, the overbooking controls, depend on several inputs: the type of distribution used to describe the cancellation process (F), the intensity of the cancellation process (q), the marginal revenue (p), the marginal denied boarding cost (h) and the effects of cancellation and new reservations over time.

To describe the cancellation process, the state of practice uses a binomial distribution with the probability of the event happening - *q(t)* dependent on the time remaining until departure *(t) (*see Equation 5-24).

$$P(Z(x) = x) = \binom{N}{x} \cdot q(t)^x \cdot \left(1 - q(t)\right)^{N-x};\ x = 1,2, \dots, N \tag{5-24}$$

$$where\ Z(x) -\ realized\ demand; N - number\ of\ bookings\ on\ hand$$

Values of marginal revenue and marginal cost used in risk base overbooking models depend on the type of model used to control available capacity and assumptions about passenger denied boarding behavior. For bid-price controls the optimal marginal revenue is determined in the context of monotonicity[39] conditions of the value function. For booking limits or protection levels controls, the heuristic marginal revenue is determined as a weighted average fare. Finally, the marginal cost is equal with the compensation offered to passenger whose boarding was denied and typically assumed to follow a convex function *V(x)* with a constant gradient (Equation 5-25).

$$V(x) = \begin{Bmatrix} 0 & \\ -h(x - C) & y > C \end{Bmatrix} \tag{5-25}$$

In the context of the dynamics of cancellations and new reservations over time, overbooking models can be static or dynamic. Overbooking limits determined with static models use estimates of cancellation proportions and bookings on hand to determine local

---

[39] The marginal revenue is decreasing in remaining capacity and increasing in time until departure – Talluri and Van Ryzin (2004a)

$\Delta V_j(x) \equiv V_j(x) - V_j$(x-1) then

$\Delta V_j(x + 1) \leq \Delta V_j(x)$
$\Delta V_{j+1}(x) \leq \Delta V_j(x)$

optimums. In contrast, overbooking limits determined with dynamic models use current and future estimation of demand and cancellation rates to determine global optimums. As approximations of dynamic models, static models can be re-solved periodically to generate close-to-optimal solutions.

With respect to what type of approach to use when simulating a general yield management problem several aspects are worth noticing. First, for exact solutions of the general yield management problem, experimental results of Subramanian, Stidham et al (1999) prove that using class-dependent cancellation rates can result in significant revenue gains. Another important finding of Subramanian, Stidham et al. is that using "close approximation" of class-dependent cancellation rates in the context of mixed dynamic programming formulations results in close to optimal results. However, the work of Subramanian, et al (1999) does not provide any methodological details on how one can determine a close approximation of class-dependent cancellation rates.

Second, for approximate solutions of the general yield management problems re-solving static overbooking models periodically remains the most popular alternative among yield management practitioners. Differences between static and dynamic formulations of risk-based overbooking models are presented in Equation 5-26 and Equation 5-27 (Talluri and Van Ryzin 2004a).

$$\bar{F}_{x-1}(C-1) \leq \frac{p}{q \cdot h} \tag{5-26}$$

$$v_{t+1}(x) = E[V_{t+1}\big(Z_t(x)\big) - \big(x - Z_t(x)\big)r(t)] \tag{5-27}$$

$$V_t(y) = E[max_{y \leq x \leq y+D_t}\{v_{t+1}(x) + (x-y)p(t)\}]$$

If, for static overbooking solutions assumptions about cancellation distribution and marginal cost and revenue suffice, for dynamic overbooking solution the intensity of the cancellation process has to be considered as well. To describe the number of surviving reservations at time t - $Z_t(x)$, both types of models use a binomial distribution with survival probability- q(t). For dynamic overbooking models time of cancellation refunds- r(t) and expected values of the new reservation requests D(t) have to be considered.

To determine the impact of time-to-event estimates of cancellation rates in the context of the general yield management problem, current simulation uses approximate solutions. The main reasons for using a heuristic instead of an exact solution have been described in Chapter 2. With the use of heuristics current dissertation establishes a lower bound on the revenue worthiness of a time-to-event cancellation forecast and limits the curse of dimensionality.

Finally, with respect to the set of assumptions used, our simulation differs from the state of practice on several aspects. For the type of distribution used to describe the cancellation process current dissertation uses a Poisson distribution. Indeed, since cancellations are analyzed from a time-to-event perspective, the intensity of the cancellation process depends on the distribution of cancellations from new bookings and not on the distribution of cancellations from bookings on hand. In this context, determining cancellation probabilities based only on time until departure leads to identification problems. Also, as a closely related alternative to the binomial distribution, the Poisson distribution has the advantage that exact knowledge about the population of risk and the probability of an event happening is not required. As such, in the case of rare events, *e.g.*, cancelled tickets, the Poisson distribution is more stable at predicting expected cancellation effects over long periods of time.

For marginal revenue and marginal cost values[40], this dissertation uses the lowest fare and the highest fare. To determine overbooking limits, the newsvendor model (Littlewood rule) is used. The expected number of cancellations is determined as the inverse of a cumulative Poisson distribution with probability equal with the critical ratio and the mean of the distribution equal with the forecast of cancellations (see Equations 5-28, 5-29). Finally, an overbooking factor is determined as the ratio between the expected number of cancellations and expected demand.

$$Critical\ Ratio = \frac{p}{p+h} \tag{5-28}$$

$$E[Cancel] = INV\ Poisson\ (Critical\ Ratio, Forecast\ Cancel) \tag{5-29}$$

5.3.4. Simulation of a single resource capacity control

To assess the impact of time-to-event forecasts on the current yield management practice, a complete simulation of a single resource capacity control (SRCC) was designed. Revenue opportunities are identified by the expected difference between revenue streams of a SRCC under a time-to-event cancellation forecast and revenue streams of a SRCC under a state-of-practice cancellation forecast.

To replicate the deployment of a new yield management system, the SRCC simulation is divided between two stages: the preliminary stage and the simulation stage. The following paragraphs describe the details of the simulation for each of these two stages.

Preliminary stage is equivalent with an initialization cycle of a new yield management system. Start-up values for the demand (mean and standard deviation) are

[40] Here, values refer to leg-defined fare buckets.

determined such they will match empirical characteristics of the ARC data sample. For each time period of the booking horizon, overbooking and seat inventory controls are re-adjusted using the Littlewood's rule and the EMSR-b heuristic. Finally, demand is unconstrained and used to update original values of mean and standard deviation.

Figure 5-3 presents the conceptual framework of the preliminary stage. To assess the impact of time-to-event cancellation forecasts on airlines revenue streams, two categories of overbooking controls (time to event -TTE and booking on hand - BOH) are applied to the same arrival stream.

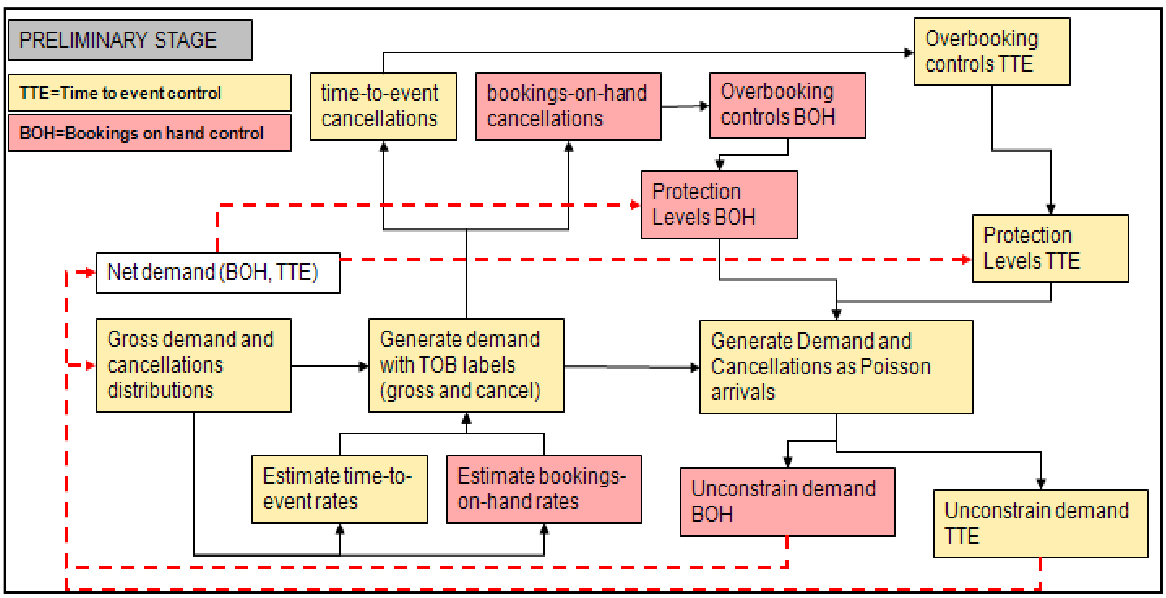

Figure 5-3: SRCC Simulation - Preliminary Stage

The first step in the preliminary stage simulation is **the initialization** of the main input values for SRCC simulation. The set of values for the capacity, the total gross demand, the fare structure, the number of booking intervals, and cancellation percentage identify the characteristics of a simulation scenario and are initialized using the following set of assumptions:

- To match current characteristics of domestic fleets, flight capacity is assumed to vary between 100 and 200 seats.
- To guarantee that capacity allocation runs under saturated demand conditions, the start-up value for the total gross demand is determined as two times the available capacity.
- Since information about fare buckets is not available, a simplified fare structure consisting of three classes with threshold[41] values equal with $100 (discount), $200 (economy) and $300 (business) is assumed.
- Booking horizon is divided into three day from departure booking intervals (0-14, 15-21, and 22-90) or six booking intervals (0-7,15-21, 22-30, 31-45, 46-60, and 61-90) booking intervals.
- Total cancellation percentage is assumed to vary between 10% and 30%.

Initialization step is followed by **the start-up step**. With the main input values defined, the SRCC simulation initializes the demand (mean and standard deviation of the gross demand and the conditional cancellation distributions) for each fare class and booking interval using characteristics of the ARC data sample. Start-up values for gross demand (mean and variance) are estimated using the assumed total gross demand and the empirical distribution of gross demand resulted from ARC data sample. The mean of gross demand is determined by multiplying the assumed total gross demand with the correspondent ARC percentage. The variance of demand is considered equal with the mean.

[41]Used to identify fare buckets, *i.e.*, $0 -$100 one-way fares are mapped to discount class, $101-$200 one way fares are mapped to economy class, >$200 one way fares are mapped to business class.

Similarly, start-up values for conditional cancellations (mean and variance) are estimated using the total assumed cancellation percentage, the total ARC dataset cancelation percentage (2.2%), and the sample hazard estimators resulted from ARC data sample. The mean of conditional cancellations is determined by multiplying the ratio between the assumed cancellation percentage and the empirical cancellation percentage with the correspondent ARC hazard estimator. The variance of demand is considered equal with the mean.

To be able to store and update the mean and the standard deviation the distributions of gross demand and conditional cancellations across multiple time dimensions (*i.e.*, time of booking and days from departure) current research uses a matrix format. Figure 5-4 presents the mean and standard deviation matrixes for a booking horizon consisting of five intervals. Both bookings with time of booking *i* and cancellations from bookings with a time of booking *i* at period *j* are assumed to be normally distributed: $N_d\ (d_i, \sigma_i^2)$ and $N_c\ (c_{ij}, \sigma_{ij}^2)$.

Initial values of gross demand and cancellations are used to determine the set of inputs for overbooking and capacity allocation controls procedures. The **estimation step** uses mean values of the distributions of gross demand and cancellations and consists of two steps. First, according to procedures describes in Section 2.2 the sample estimates of the conditional probabilities of cancelling (hazards) - h_{ij} and of the state of practice cancellation probabilities (rates) - r_j are determined. Second, start-up values of the mean nd_{ij} and variance of the net demand distribution are estimated.

Mean - Gross Demand and Cancellations

Demand	Cancel_5	Cancel_4	Cancel_3	Cancel_2	Cancel_1
d_5	c_{55}	c_{54}	c_{53}	c_{52}	c_{51}
d_4		c_{44}	c_{43}	c_{42}	c_{41}
d_3			c_{33}	c_{32}	c_{31}
d_2				c_{22}	c_{22}
d_1					c_{11}

Standard Deviation – Gross Demand and Cancellations

Demand	Cancel_5	Cancel_4	Cancel_3	Cancel_2	Cancel_1
σ_5	σ_{55}	σ_{54}	σ_{53}	σ_{52}	σ_{51}
σ_4		σ_{44}	σ_{43}	σ_{42}	σ_{41}
σ_3			σ_{33}	σ_{32}	σ_{31}
σ_2				σ_{22}	σ_{22}
σ_1					σ_{11}

Figure 5-4: Time of Booking Data Storage

Following the estimation stage, **the forecasting step** generates future values of gross demand - $F(d_i)$ as random realizations of current distributions. To maintain consistency across time-to-event and booking-on hand streams, forecasts of booking on hand - $F(b_i)$ result as a difference between cumulative values of time-to-event gross demand and cumulative values time-to-event cancellations. Cancellations are computed using the set of cancellation probabilities (h_{ij} and r_i) determined in the estimation stage and forecasts of gross demand and bookings on hand. Forecasts of cancellations at time period j from bookings with a time a booking i - $F(c_{ij})$ result from multiplying the forecasts of gross demand $F(d_i)$ with the estimates of cancellation hazards - h_{ij}. Forecasts of cancellation at time period j form bookings on hand results from multiplying the forecasts of bookings on hand - $F(b_i)$ with the estimates of cancellation rates - r_j.

In **the control step,** forecasts combined with the estimates of net demand and cancellation probabilities are used to generate the arrival stream for the current run of

SRCC simulation and to determine the set of controls (overbooking and capacity allocation). To isolate the effect of overbooking controls on revenue streams, the actual values of gross demand are considered equal with forecasts of gross demand. In contrast, the actual values of cancellations are generated as random realization of the cancellation distributions - CC.

Simulated arrival streams result from the combination of actual gross demand and actual cancellations, each distributed across booking intervals according to Poisson arrivals. To facilitate the update of simulation statistics and ensure consistency when processing arrival streams, a time-of-booking label is associated with each booking request and cancellation event.

Overbooking levels for each booking interval are estimated using the following steps:

- Compute the mean of the cancellation distribution as a sum of conditional cancellations forecasts - $F(c_{ij};\ _{j=n,..i})$.
- Use the risk based algorithm defined in Section 5.3.2 to determine the maximum number of allowed cancellations.
- Divide the maximum number of cancellation by forecasts of gross demand and bookings on hand to determine cancellation proportions.
- Compute the inverse of the survival proportion to determine overbooking factors.
- Use estimates of overbooking factors multiplied by available capacity to determine overbooking levels (*i.e.*, the maximum number of allowed reservations at a certain point in time).

To determine capacity allocation controls a dynamically adjusted EMSR-b heuristic is used (see Section 5.3.1). At each booking interval, estimates of expected future net demand for each fare class and overbooking levels are used to determine protection levels.

Using current values of the set of controls (overbooking and protections levels), **the processing step** generates a sequence of accept/reject decisions for booking requests. In this context, the state of practice for processing arrival streams was adjusted to take into account the fact that demand streams are generated in a multi-dimensional context (*i.e.*, time-of-booking and days from departure). First, simulation statistics (the number of accepted requests, the number of denied requests, cancellations, the net demand, and the available capacity) are recorded across both time dimensions. Second, demand streams are processed across both sets of overbooking controls (time-to-event and bookings on hand).

In the context of a defined arrival stream, the processing procedure manages two sets of decisions: (1) accept or reject a booking request, and (2) process or not process a cancellation. With respect to the first category, a booking request is accepted if and only if the available capacity is greater or equal to one. If available capacity is greater than one, a booking request is accepted provided that remaining capacity is greater or equal with the protection level of immediately higher class.

With respect to the second category of decisions, a cancellation for a fare class is processed if and only if there are available reservations in that class. To ensure consistency of arrival streams, the availability of reservations is checked across the time of booking dimension, *i.e.*, cancellations at booking interval j from bookings with time of booking *i* can only occur if the total number of reservations with time of booking *i* is greater or equal to one.

To avoid the spiral-down effect of revenues (Talluri and Van Ryzin 2004a), resulted values of gross demand and cancellations need to be unconstrained. **The unconstraining step** uses the same level of detail as the processing step. Resulted streams of gross demand and cancelations are unconstrained at a time-of-booking level. As a final stage of the preliminary stage simulation, **the updating step** uses unconstrained values of demand and cancellations to update the input matrixes. Finally, to allow the values of gross demand and cancellations to stabilize, all steps of the preliminary stage were repeated for 15 times.

As the second stage of the simulation, **the simulation stage** is equivalent with the production cycle of a revenue management system. Since input values were stabilized during the "warm-up" stage, the revenue streams of the two simulation scenarios (time-to-event and bookings on hand) can be recorded and compared. Figure 5-5 presents the conceptual framework of the simulation stage.

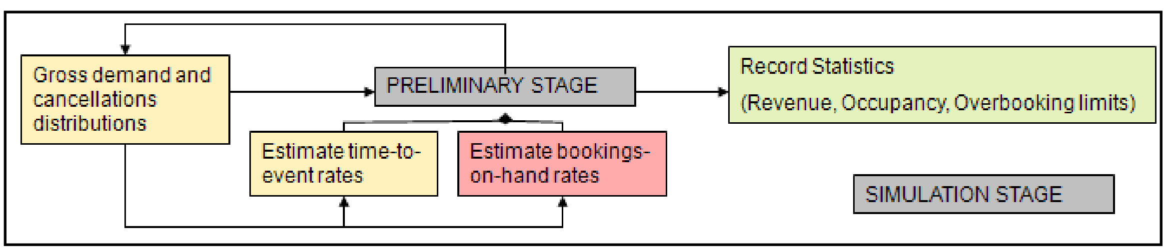

Figure 5-5: Single Resource Capacity Control Simulation – Simulation Stage

Each run of the simulation stage re-estimates the time-to-event rates and the bookings-on-hand rates using updated values of gross demand and cancellations. These estimates are used as inputs to the preliminary stage cycle. To compare the performance of time-to-event forecasts with bookings-on-hand forecasts, simulation statistics are recorded and compared. For each scenario, the production cycle is repeated 100 times.

Chapter 6: RESEARCH RESULTS

This chapter sections presents research results. As an alternative to time-to-event models, estimation results of a MNL model are presented Section 6.1. Results of time to event analysis are presented in Section 6.2 and organized in several subsections: exploratory analysis, DTPO cancellation hazard model, competing risk model, and validation of DTPO model. Section 6.3 presents results of the Single Resource Capacity Control (SRCC) simulation.

6.1. A Multinomial Logit Model of Ticket Exchanges and Refunds

As an alternative to time-to-event models, a multinomial logit model of ticket exchanges and refunds is estimated at a leg level. Using a sample of 6,204 ticketing records from the ARC data, MIA-SEA market, the effect of carrier, trip and booking characteristics on ticket exchanges and refunds is explored.

In the context of polychotomous models several hypothesis are tested. First, compared to smaller carriers (market shares less than 10%) major carriers[42] (*i.e.*, carriers with market shares higher than 10%) are expected to exhibit higher exchange and refund probabilities. Second, due to higher frequency of rescheduling activities, tickets booked closer to flight departure are expected to exhibit higher refund and exchange probabilities. Third, with lower price sensitivity and higher flexibility of associated fare rules, higher fare classes are expected to exhibit higher refund and exchange probabilities. Fourth, since one way tickets are typically associated with

[42] For the considered markets major carriers are synonymous with legacy carriers

business travelers we expect them to exhibit higher refund and exchange probabilities when compared to round tickets.

The dependent variable in the multinomial logit model (MNL) is defined by passengers' choice of exchanging, cancelling, or keeping the original ticket. If **P(*y* = *m* | *x*)** is the probability of observing outcome ***m*** given characteristics ***x*** then the probability model for ***y*** is defined in Equation 6-1.

$$\Pr(y_i = m \mid x_i) = \frac{\exp(x_i \beta_m)}{\sum_{j=1}^{J} \exp(x_i \beta_j)} \tag{6-1}$$

For "no event" or "keep the original ticket" as reference category, MNL estimation results are presented in Table 6-1. Results of MNL model indicate a good data fit and are generally consistent with the hypothesized scenarios. As expected, passengers flying on major carriers are more likely to cancel or exchange their tickets relative to minor carriers. Relative to the reference category of all other carriers (defined as carriers with market share less or equal then 10%), both major carriers have higher probabilities of exchanges and cancellations, and higher frequencies of exchange behavior then cancellation behavior. These findings may capture market-specific characteristics such as the variety of itinerary and ticketing options provided by a major carrier, which is expected to be more important for business travelers. Moreover, to the extent that business travelers are more likely to select major carriers with frequent flight departures (that represents more re-scheduling opportunities), this result is consistent with hypothesized expectation that both cancellations and exchange rates would be higher for major carriers.

In terms of trip characteristics, it was expected that both cancellation and exchange probabilities would be higher for one-way fares. However, results indicate that only cancellation probabilities are higher for one way fares. Since we suspect that this is due to data misspecification[43] we excluded the effects of one-way versus round trips from future analysis.

Table 6-1: MNL Model of Exchange and Cancellation Choice for Airline Tickets

Parameter	choice	DF	Standard Estimate	Wald Error	Chi-Square	Pr > ChiSq
Intercept	2	1	-6.8331	0.6381	114.6837	<.0001
Intercept	1	1	-1.9375	0.1452	178.0680	<.0001
Carrier9	2	1	1.1145	0.5358	4.3271	0.0375
Carrier9	1	1	0.4363	0.1269	11.8129	0.0006
Carrier8	2	1	1.9554	0.5067	14.8949	0.0001
Carrier8	1	1	0.5529	0.1192	21.5282	<.0001
RoundTrip	2	1	0.8031	0.3761	4.5601	0.0327
RoundTrip	1	1	-2.2784	0.1437	251.2181	<.0001
BookCurve0to3	2	1	-1.8530	0.6120	9.1675	0.0025
BookCurve0to3	1	1	-0.6664	0.1525	19.0916	<.0001
BookCurve4to7	2	1	-1.6347	0.6139	7.0915	0.0077
BookCurve4to7	1	1	-0.1745	0.1587	1.2097	0.2714
BookCurve8to14	2	1	-0.1737	0.3845	0.2042	0.6514
BookCurve8to14	1	1	0.0749	0.1549	0.2340	0.6286
BookCurve15to21	2	1	-0.00624	0.3818	0.0003	0.9870
BookCurve15to21	1	1	0.1631	0.1772	0.8472	0.3573
BookCurve22to30	2	1	-0.3356	0.4346	0.5961	0.4401
BookCurve22to30	1	1	0.0103	0.1989	0.0027	0.9588
fare_ow	2	1	0.00240	0.000675	12.6290	0.0004
fare_ow	1	1	0.000206	0.000263	0.6150	0.4329

No observations	6204
Log-likelihood at 0 - L(0)	-2555.712
Log Likelihood at convergence - L(β)	-1779.262
-2(L(0)- L(β))	1552.9
R-Square	0.2214
Max-rescaled R-Square	0.3945

Choice set: 2 - Exchange , 1-Cancel 0-Normalizing alternative (No event)

The impact of time of booking on exchanges and cancellations is significant only for short-term horizon bookings (*i.e.*, BookingCurve0to3 and BookingCurve4to7). The results indicate that relative to tickets that are purchased eight or more days from flight departure,

[43] A significant number of tickets coded with round trip indicators had the outbound departure date missing.

passengers are less likely to exchange or cancel their tickets. Furthermore, the difference between the two choices overwhelmingly favors cancellation behavior over the exchange behavior. Intuitively, this result makes sense due to the underlying fare structure of airlines in this particular short period of time. Specifically, tickets purchased within the seven days of flight departure are generally higher fares that are refundable.

Finally, the results indicate that as fare increases, exchange and cancellation are more likely to occur. This is not surprising, as higher fares are typically associated with an increase in the availability of exchange and cancellation opportunities. To the extent in which higher fares are purchased by business travelers, the results are consistent with hypothesized scenario.

6.2. Time-to-event Models

Results of time to event analysis are structured in several sections. To motivate the use of a discrete functional form for the base line hazard, the first section presents the results of the exploratory analysis. The second section uses exploratory analysis results to estimate a Discrete Time Proportional Odds model for the intensity of the cancellation process (*i.e.*, cancellation hazard). The last section presents goodness of fit the DTPO model compared with a series of binary logits.

6.2.1. Exploratory Analysis for Base Line Hazard

Exploratory analysis focuses on finding the most appropriate specification for the base line hazard. Using the ARC ticketing data presented in Section 4.2 sample estimators of baseline hazard with associated 95% CI and interval ticket cancellations with associated lost to follow up tickets are estimated (see Figure 6-1 and Table 6-2). Figure 6-1 points out that the intensity of cancellations decreases with days from issue.

Table 6-2**:** Interval Ticket Cancellations and Lost to Follow up Tickets

Day from Issue		Total n_i	Cancel c_i	Follow-up f_i	Survival prob.	Day from Issue		Total n_i	Cancelled c_i	Follow-up f_i	Survival prob.
0	**1**	154,367	37	2,966	0.9998	**46**	**47**	22,896	6	1,044	0.9612
1	**2**	151,364	121	6,299	0.9989	**47**	**48**	21,846	8	936	0.9608
2	**3**	144,944	188	4,772	0.9976	**48**	**49**	20,902	11	1,035	0.9603
3	**4**	139,984	136	5,236	0.9966	**49**	**50**	19,856	13	972	0.9596
4	**5**	134,612	164	4,862	0.9954	**50**	**51**	18,871	12	897	0.959
5	**6**	129,586	179	4,692	0.994	**51**	**52**	17,962	11	821	0.9584
6	**7**	124,715	182	4,034	0.9925	**52**	**53**	17,130	5	883	0.9581
7	**8**	120,499	204	5,636	0.9908	**53**	**54**	16,242	9	715	0.9576
8	**9**	114,659	150	3,632	0.9895	**54**	**55**	15,518	3	699	0.9574
9	**10**	110,877	122	3,172	0.9884	**55**	**56**	14,816	7	703	0.9569
10	**11**	107,583	97	3,260	0.9875	**56**	**57**	14,106	7	646	0.9564
11	**12**	104,226	101	3,053	0.9865	**57**	**58**	13,453	3	694	0.9562
12	**13**	101,072	97	3,054	0.9855	**58**	**59**	12,756	3	644	0.956
13	**14**	97,921	102	2,999	0.9845	**59**	**60**	12,109	5	609	0.9556
14	**15**	94,820	121	4,335	0.9832	**60**	**61**	11,495	3	561	0.9553
15	**16**	90,364	70	3,360	0.9824	**61**	**62**	10,931	1	469	0.9552
16	**17**	86,934	63	2,821	0.9817	**62**	**63**	10,461	7	517	0.9546
17	**18**	84,050	63	3,029	0.981	**63**	**64**	9,937	3	518	0.9543
18	**19**	80,958	66	2,825	0.9802	**64**	**65**	9,416	5	551	0.9538
19	**20**	78,067	51	2,695	0.9795	**65**	**66**	8,860	6	517	0.9531
20	**21**	75,321	79	2,773	0.9785	**66**	**67**	8,337	2	485	0.9529
21	**22**	72,469	93	3,061	0.9772	**67**	**68**	7,850	3	418	0.9525
22	**23**	69,315	58	2,562	0.9763	**68**	**69**	7,429	6	405	0.9517
23	**24**	66,695	45	2,574	0.9757	**69**	**70**	7,018	2	385	0.9514
24	**25**	64,076	32	2,467	0.9752	**70**	**71**	6,631	2	413	0.9511
25	**26**	61,577	35	2,345	0.9746	**71**	**72**	6,216	2	417	0.9508
26	**27**	59,197	34	2,258	0.974	**72**	**73**	5,797	1	403	0.9506
27	**28**	56,905	41	2,157	0.9733	**73**	**74**	5,393	1	399	0.9505
28	**29**	54,707	42	2,072	0.9726	**74**	**75**	4,993	3	364	0.9499
29	**30**	52,593	30	2,191	0.972	**75**	**76**	4,626	2	357	0.9494
30	**31**	50,372	36	2,251	0.9713	**76**	**77**	4,267	1	322	0.9492
31	**32**	48,085	29	2,158	0.9707	**77**	**78**	3,944	3	339	0.9484
32	**33**	45,898	19	2,060	0.9703	**78**	**79**	3,602	2	313	0.9479
33	**34**	43,819	31	1,851	0.9696	**79**	**80**	3,287	1	335	0.9476
34	**35**	41,937	35	1,808	0.9687	**80**	**81**	2,951	1	335	0.9473
35	**36**	40,094	38	1,702	0.9678	**81**	**82**	2,615	1	281	0.9469
36	**37**	38,354	27	1,549	0.9671	**82**	**83**	2,333	0	258	0.9469
37	**38**	36,778	18	1,671	0.9666	**83**	**84**	2,075	1	284	0.9464
38	**39**	35,089	32	1,539	0.9657	**84**	**85**	1,790	2	309	0.9452
39	**40**	33,518	13	1,613	0.9653	**85**	**86**	1,479	0	283	0.9452
40	**41**	31,892	22	1,530	0.9647	**86**	**87**	1,196	0	260	0.9452
41	**42**	30,340	19	1,626	0.964	**87**	**88**	936	0	255	0.9452
42	**43**	28,695	29	1,704	0.963	**88**	**89**	681	0	240	0.9452
43	**44**	26,962	14	1,310	0.9625	**89**	**90**	441	0	230	0.9452
44	**45**	25,638	15	1,457	0.9619	**90**	**91**	211	0	211	0.9452
45	**46**	24,166	13	1,257	0.9614						

Also, for tickets booked well in advance of departure date the sample hazard exhibits a higher variability. Table 6-2 results point out the need to use a hazard model formulation which accounts for the effect of lost to follow up (*i.e.*, differential chances of being at risk of cancelling or the effect of time of booking).

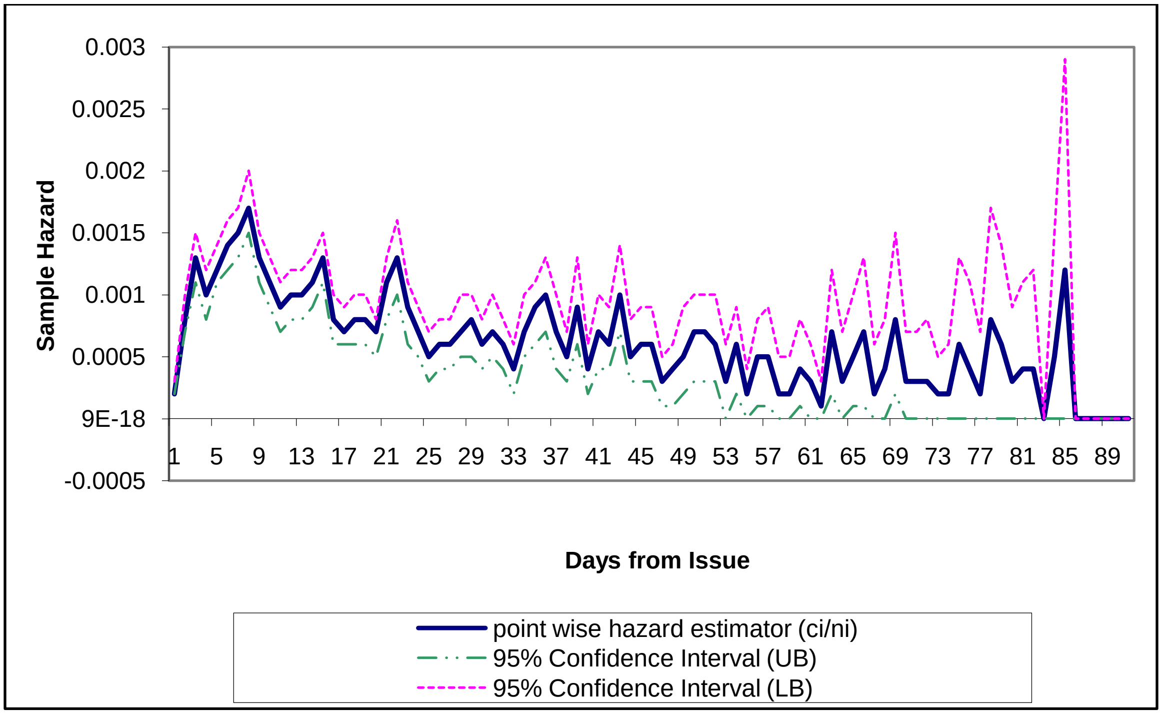

Figure 6-1: Non Parametric Hazard Estimates and 95% Confidence Intervals

Figure 6-2 presents the survival Kaplan-Meier estimator and the point-wise estimate hazard $h_j = s_j / n_j$ (where s_j – number of cancelled tickets during the j^{th} day from issue; n_j – number of total tickets during the j^{th} day from issue). To explore possible candidates for the baseline hazard, a lowess smoother with neighborhood bandwidth equal to 0.1 is associated with the point-wise estimators of sample hazard.

With the exception of the 0-3 days from issue (DFI) time interval, the visual inspection of Figure 6-2 reinforces the idea of a decrease of cancellation hazard with days from issue. However, the hazard decrease is not strictly monotonic, and one can observe large jumps in hazard values around 3, 7, 14 and 21 DFI combined with constant hazards for 21-45 and 46-80 DFI time intervals.

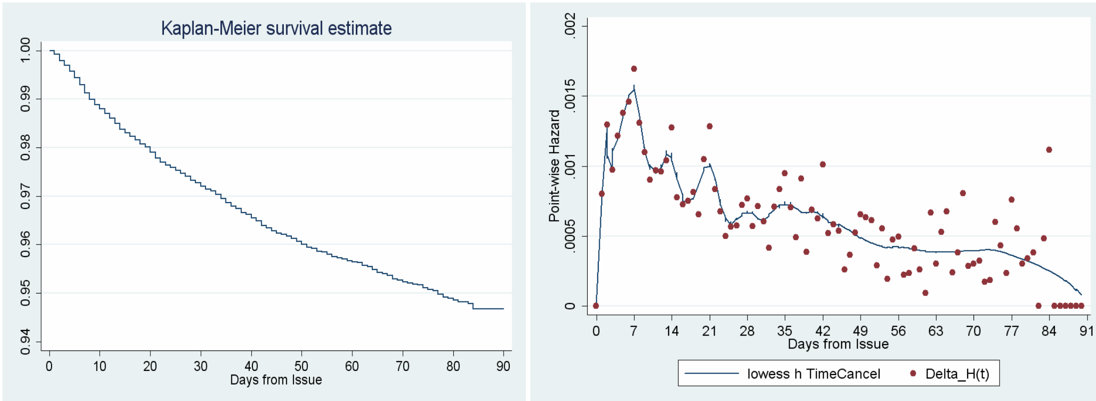

Figure 6-2: Non-Parametric Estimators for Survival and Hazard Functions

Lower values of the cancellation hazard in the 0-3 DFI time period relative to the 4-7 DFI time period are somehow surprising. In analyzing this phenomenon, one has to consider the way in which population at risk is considered. For example, the population at risk for the 0-3 DFI time period includes the entire mix of business and leisure passengers. In this context, without an indicator to separate tickets cohorts based on their time of booking the effects of business versus leisure are difficult to isolate.

One possible explanation for the hazard decrease is the fact that business passengers purchasing very close to departure (present only in the DFI 0-3 period) are more certain of their travel plans and thus less likely to cancel and exchange tickets. Another explanation is the stricter set of rules associated with tickets purchased in the near proximity of departure date. Regardless of the reason, Figure 6-2 points out that the development of the cancellation hazard over time results from a combination of time of booking and days from issue (or days from departure and days from issue) effects. The hazard jumps at 3, 7, 14 and 21 DFI followed by constant values for 21-45 and 46-80 DFI time intervals are consistent with refund and exchange rules associated with advance purchase policies. Indeed, all classes of refundable tickets are entitled to refunds provided changes are made until advance purchase deadlines.

6.2.2. Cancellation Hazard - Discrete Time Proportional Odds (DTPO) Model

Findings of Section 6.2.1 were used as a basis to define three baseline hazards specifications for the DTPO model: linear, logarithmic and discrete ((0-3], (3-7], (7-14], (14-21], (21-45] and (45-90]). Estimation results for the transformed ARC dataset using these specifications are presented in Table 6-3. The likelihood ratio test and non-nested hypothesis tests were used to statistically compare the fit of the different models and select the discrete formulation as the preferred specification. Formally, the likelihood ratio test rejects the null hypothesis that the linear and discrete models are equal at a 0.001 significance level since -2 $LL_R - LL_U >> \chi^2_{4,0.001}$, or 66.7>>18.5. Likewise, the non-nested hypothesis test (Horowitz 1982) rejects the null that the logarithmic and discrete models are equal; the significance of the decision rule for this test is given by Φ(-12.64) = << 0.001.

Table 6-3**:** Comparison of Baseline Hazard Specifications

Baseline hazard functional form Ψ_{ij}	*Parameter Estimates*	*Log Likelihood*	*Pseudo-R2*
$Discrete: \beta_{0\_3} \cdot d_{0\_3} + ... + \beta_{21\_45} \cdot d_{21\_45} + \alpha_{45\_90}$	$\beta_{0\_3} = 0.981 \quad \beta_{4\_7} = 1.240$ $\beta_{8\_14} = 0.959 \quad \beta_{15\_21} = 0.723$ $\beta_{22\_45} = 0.470 \quad \beta_{46\_90} = -7.778$	-26,686.20	0.0071
$Logarithmic: \alpha + \beta \cdot \ln(t)$	$\alpha = -6.419 \quad \beta = -0.236$	-26,764.74	0.0042
$Linear: \alpha + \beta \cdot t$	$\alpha = -6.634 \quad \beta = -0.020$	-26,719.56	0.0059

The estimation results of the Discrete Time Proportional Odds model are presented in Table **6-4** (odds-ratio format). We report an adjusted R-square of the

DTPO model equal with 0.057 and significant effects of all considered covariates on the cancellation hazard.

Table 6-4: Discrete Time Proportional Odds Estimation Results

Covariates			*[95% Confidence Interval]*	
	Parameter	*z-stat*	*lower bound*	*upper bound*
Time (DFI=Days from Issue; reference category 46-90 days from issue)				
DFI_0_3	2.135	8.19	1.780	2.559
DFI_4_7	2.966	12.32	2.495	3.526
DFI_8_14	2.461	10.36	2.075	2.918
DFI_15_21	2.048	7.88	1.714	2.448
DFI_22_45	1.681	5.98	1.417	1.993
Days from Departure (DFD)				
DFD	0.963	-24.22	0.960	0.966
Group Size (reference= one person)				
2 people	0.440	-13.93	0.392	0.494
3 or more people	0.304	-10.88	0.245	0.377
Saturday Night Indicator				
Saturday night	0.779	-5.66	0.714	0.849
Outbound Day of the Week (reference = Sunday)				
Monday	1.297	4.33	1.153	1.460
Tuesday	1.275	3.86	1.127	1.442
Wednesday	1.135	1.99*	1.002	1.287
Thursday	0.862	-2.14	0.753	0.987
Friday	0.823	-2.74	0.716	0.946
Saturday	0.945	-0.71*	0.809	1.104
Market (reference =Bos-Mia)				
Bos-Sea	0.653	-7.69	0.586	0.728
Hnl-Ord	0.441	-5.69	0.333	0.585
Mia-Bos	0.618	-7.28	0.543	0.703
Mia-Sea	1.347	3.86	1.158	1.567
Ord-Hnl	0.669	-4.85	0.569	0.788
Sea-Bos	0.629	-7.61	0.558	0.709
Sea-Mia	0.625	-5.86	0.535	0.732
Carriers (masked information)				
Carrier 2	1.133	2.20	1.014	1.266
Carrier 3	0.392	-10.79	0.331	0.465
Carrier 4	0.804	-2.20	0.662	0.977
Carrier 5	1.089	1.20*	0.948	1.250
Pro-Rated Fare				
Fare	1.001	20.35	1.001	1.002
Goodness of fit statistics				
Number of obs.	151,401	(equivalent of 3,707,425(day-person observations)		
LR chi2(df)		3100.41 (27)		
Pseudo R^2		0.0577		
Log likelihood		-25327.923		

Note: Z-stats are reported against estimated coefficients (not shown).

The highly significant coefficient of *days from departure* indicates that cancellation rates depend on how close passengers are from their departure date. As expected, the coefficient shows a decrease in the odds of cancelling by a factor of 0.96, with each extra day from departure suggesting a strong effect of the "last-minute" change of plans on cancellations. Given cancellation rates are influenced both by the time from ticket purchase (issue) and the time from departure, results from this study reinforce previous empirical evidence (Westerhof 1997) on the violation of the memoryless property.

In view of currenlty methods used to forecast cancellation rates, this finding is particularly important. Specifically, it suggests that determining cancellation proportions only as extrapolations of previously realized values[44] may not be valid, as different cancellation proportions will be observed depending on when a passenger tickets (see, Figure 6-3).

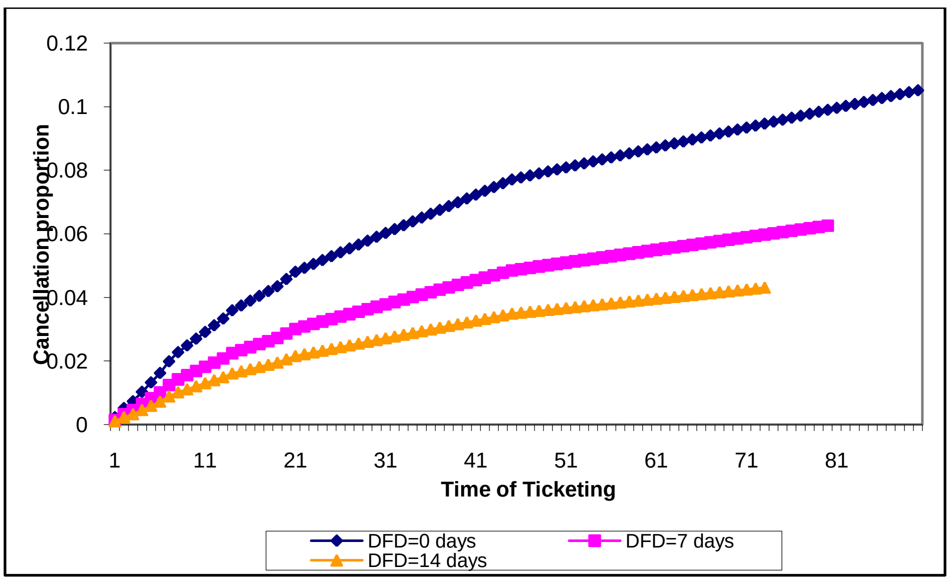

Figure 6-3: Evidence on the Violation of the Memoryless Property

Several other covariates were also examined in the study, including the outbound departure day of week, presence of a Saturday night stay on the itinerary, group size,

[44] The use of separate cancellation rates for each booking class only partially corrects for this problem, as some booking classes are available for purchase over the entire (or large portion) of the booking horizon.

carrier, market, and pro-rated fare. These variables typically associated with leisure passengers exhibit decreased cancellation rates. For passengers with a Saturday night stay, the odds of cancelling decrease by a factor of 0.78 relative to those without a Saturday night stay. In addition, the odds of cancelling for passengers traveling in groups decrease by factors of 0.30 to 0.44 relative to passengers traveling alone. Those traveling with two or more individuals are less likely to cancel than those traveling with just one other person. Finally, the odds of cancelling for passenger traveling outbound early in the work week (typically associated with business travelers) are higher than those departing later in the week. Specifically, relative to the reference category (Sunday), the odds of cancelling for Monday, Tuesday and Wednesday departures increase by factors of 1.30, 1.28 and 1.14, respectively, while the odds of cancelling for Thursday, Friday and Saturday decrease by factors of 0.86, 0.82 and 0.95, respectively.

The effects of the last three categories of covariates: *Market*, *Carrier*, and the *Pro-Rated Fare*, although significant, are more difficult to generalize because of endogenity concerns (the fare is highly correlated with market, and different carriers may impose different refund and exchange ticketing policies).

6.3. DTPO Model Validation – Receiver Operating Characteristics Analysis

The purpose of current validation procedures is to compare predictive performance of a Discrete Time Proportional Odds model to a series of binary logit models. In this context, the two categories of models are ranked across two validation procedures: (1) the likelihood of cancellation, and (2) predicted number of cancellations.

With respect to the first category, the DTPO model and the series of binary logit models are ranked in the ability to predict the propensity of the cancellation process, *i.e.*

the conditional probability of cancelling tickets. With respect to the second category, the DTPO model and the series of binary logit models are compared in the ability to predict future number of cancellations. To generate model predictions, the ARC data are divided into estimation (75% of the data) and holdout (25% of the data) samples. Also, to ensure unbiasness of model estimators observations from holdout sample are randomly selected.

To assess the ability of models to correctly predict cancellation hazards, Receiver Operating Characteristics (ROC) analysis is used. For both categories of models, maximum likelihood coefficients from estimation data sample are used to predict conditional cancellation probabilities of the holdout data sample. For the DTPO model, this procedure is used only once. For the second category of models, a binary logit model is estimated for each of the time intervals of the DTPO model described in Section 6.2. Estimation results for the two categories of models are presented in Appendix B. As a performance indicator of the two categories of models Figure 6-4 presents the ROC area plots.

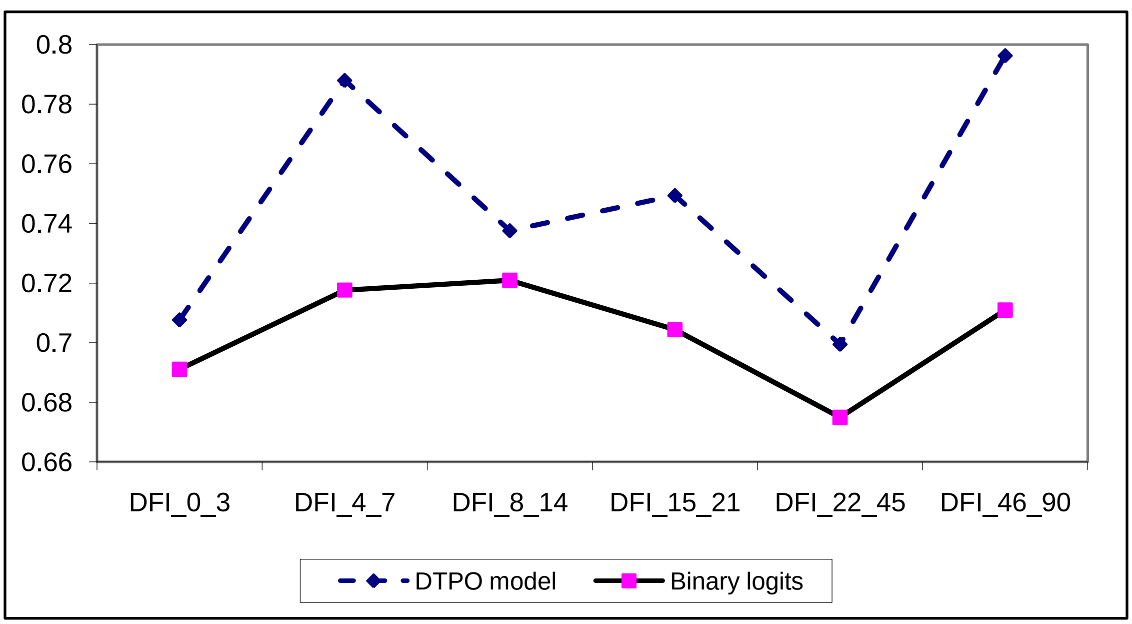

Figure 6-4: ROC Areas for DTPO and Binary Logit Models

For each of the time intervals, ROC areas are determined as the total sum of predicted probabilities of passengers in the hold-out sample that cancelled tickets (true positive rate). When compared to a series of binary logit models the DTPO model ranks superior in the ability to predict the propensity of the cancellation process for all time intervals in the booking horizon.

To compare models predictive performance in terms of number of cancellations, predicted probabilities are summed over the set of available observations in the hold-out data. Figure 6-5 presents predicted number of cancellations for each time interval. Although differences between the two categories of models are very small, the DTPO model prediction performance is superior to a series of binary logits.

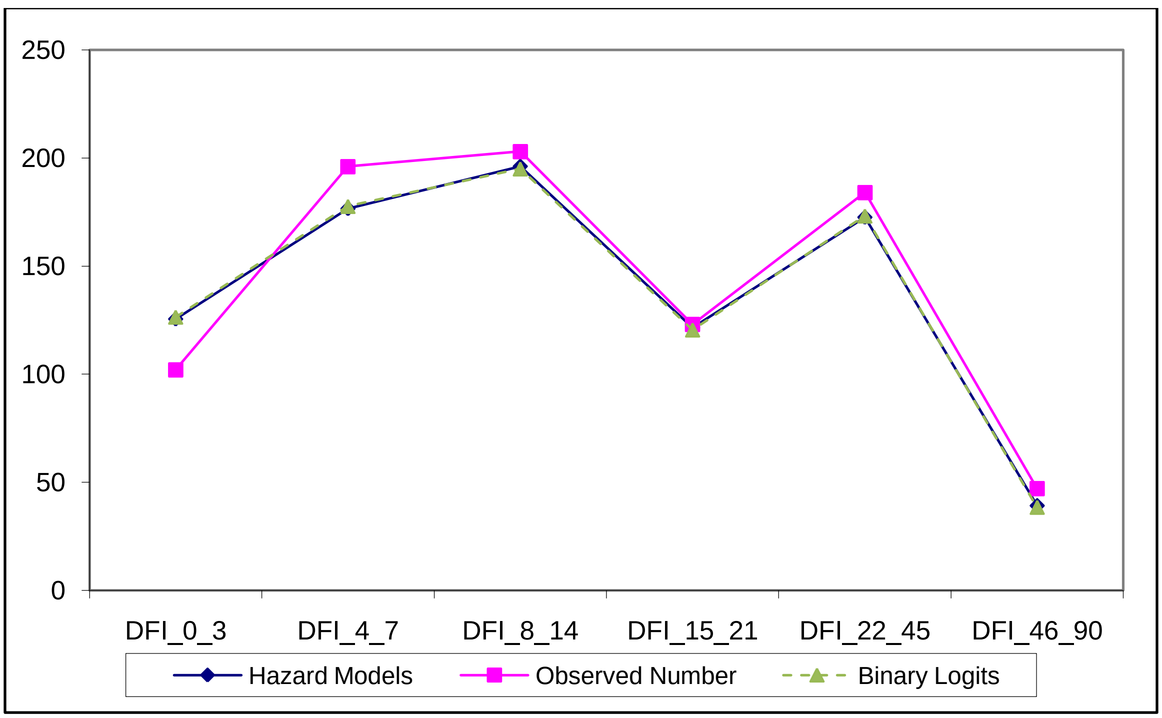

Figure 6-5: Number of Cancellations for DTPO and Binary Logit Models

Overall, model validation results indicate that when compared to a series of binary logits the DTPO model does a better job of explaining and predicting the propensity of passengers' to cancel tickets.

6.4. Single Resource Capacity Control Simulation Results

In the context of time-to-event and bookings-on-hand cancellations forecasts Table 6-5 and Table 6-6 present the results of the SRCC simulation. Using procedures described in Section 5.3.4, revenues for 16 x 2 scenarios are estimated and compared. Estimation results prove that when compared with current state of practice, time-to-event cancellation forecasts typically generate additional revenues. Minimum revenues uplifts vary from -0.24% to 3.33% while maximum revenue uplifts vary from 0.27% to 9.06% (see Figure 6-6).

Table 6-5: Revenues for Time-to-Event (TTE) Streams

			Revenue BOH		95% confidence intervals	
CAP	Per	N_BI	*Mean*	*Variance*	*lower*	*upper*
100	10	3	17,914	8,937,580	12,054	23,774
100	20	3	17,512	5,232,178	13,029	21,995
100	30	3	18,342	4,984,481	13,966	22,718
150	10	3	29,717	14,314,557	22,301	37,133
150	20	3	28,058	10,624,077	21,669	34,447
150	30	3	24,633	7,400,819	19,301	29,965
200	10	3	38,709	18,424,666	30,296	47,122
200	20	3	34,535	10,941,894	28,052	41,018
200	30	3	34,114	11,573,135	27,446	40,782
100	10	6	21,412	4,398,844	17,301	25,523
100	20	6	19,605	3,387,955	15,997	23,213
150	10	6	30,581	5,482,363	25,992	35,170
150	20	6	29,894	6,422,994	24,927	34,861
150	30	6	26,862	2,060,360	24,049	29,675
200	10	6	38,013	9,426,193	31,995	44,031
200	20	6	41,067	6,701,223	35,993	46,141

Table 6-6: Revenues Uplifts from Bookings on Hand (BOH) Streams

			Difference Revenue		95% confidence intervals	
CAP	**Per**	**N_BI**	*Mean*	*Variance*	*lower*	*upper*
100	10	3	244	3,978	120	368
100	20	3	957	13,033	733	1,181
100	30	3	832	13,682	603	1,061
150	10	3	764	8,179	587	941
150	20	3	1,256	8,741	1,073	1,439
150	30	3	968	29,890	629	1,307
200	10	3	490	6,249	335	645
200	20	3	1,049	19,824	773	1,325
200	30	3	958	19,632	683	1,233
100	10	6	-1	944	-61	59
100	20	6	69	1,110	4	134
150	10	6	4	1,151	-63	71
150	20	6	207	2,602	107	307
150	30	6	261	7,571	90	432
200	10	6	16	2,081	-73	105
200	20	6	1,039	8,514	858	1,220

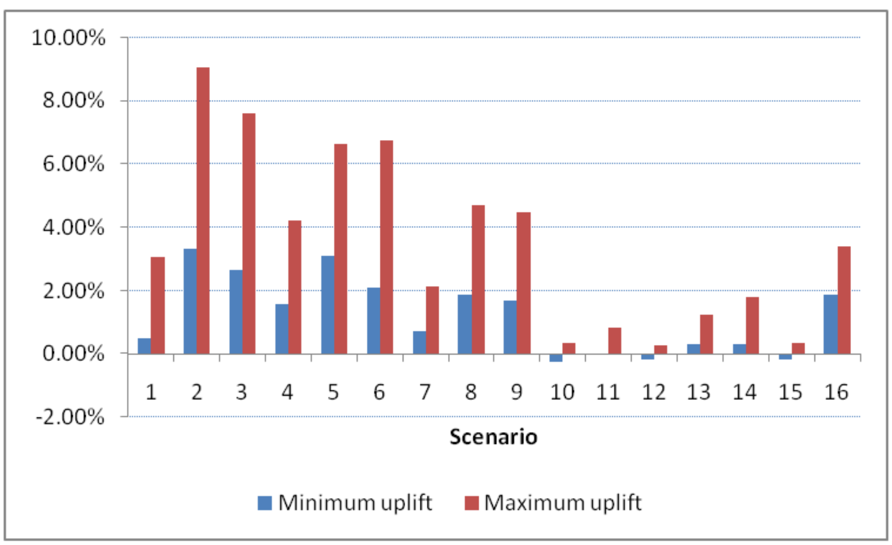

Figure 6-6: Revenue Uplifts for Time to Event Forecasts

To better understand the role of input parameters in generating revenue uplifts in the context of time-to-event (TTE) cancellation forecasts, Table 6-7 present the result of a linear regression model with the dependent variable equal with the percentage revenue increase.

Table 6-7: Uplift from Time-to-Event Cancellation Forecast (Linear Regression)

	Coefficients	*Std. Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0.08330	0.02175	3.830	0.002	0.036	0.131
Capacity	-0.00012	0.00010	-1.211	0.249	0.000	0.000
Cancellation Percentage	0.00134	0.00050	2.706	0.019	0.000	0.002
Booking Intervals	-0.01285	0.00260	-4.942	0.000	-0.019	-0.007
R Square	0.77					
Adjusted R Square	0.71					

We report an adjusted R-square of 0.77 and significant coefficients for the cancellation percentage and the number of booking intervals. As expected, the effects of cancellation percentage and booking intervals on the revenue generated by time-to-event cancellation forecasts are opposite. For one unit increase in cancellation percentage, the percentage increase in revenue from a time-to-event cancellation forecasts is equal with 0.00134. In contrast, for one unit increase in the number of booking intervals the percentage decrease in revenue from a time-to-event cancellation forecast is equal with 0.01285. These findings point out that time-to-event cancellation forecasts are particularly powerful for carriers with a simple fare structure and relatively simple advance purchase structure[45].

Surprisingly, under time-to-event cancellation forecasts, capacity does not appear to have a significant impact on revenue increases. This finding contradicts empirical

[45] Low cost carriers or regional jets are good candidates.

examples of Subramanian et al. (1999) that show that "when we have the possibility of cancellations the function u_n and the optimal policy depend on the total capacity, C, and the capacity remaining s=C-x." This result is part due to the way in which the general yield management problem was approached (*i.e.*, heuristic versus optimal) and due in part to conceptual differences between time-to-event forecasts and bookings-on-hand forecasts.

Chapter 7: CONTRIBUTIONS AND FUTURE RESEARCH

There are four main contributions of this work. First, in the context of the general yield management problem it presents an updated literature review of cancellation forecast practice. Differences between pre-deregulation and post-deregulation era are described and empirical evidence which question properties of cancellation probability are discussed.

Second, in order to decide what type of data to use to determine transitional properties of the cancellation process, ticketing and booking data sources are contrasted. As an alternative to the current state of practice, this research uses the Airline Reporting Corporation (ARC) ticketing data. Compared with previous data sources, the ARC data permits the analysis of the cancellation process from a multi-market multi-carrier perspective and ties directly to the financial streams of carriers.

Third, compared to cancellation models reported in the literature or used in practice, this research is the first to motivate and use more "customer-focused" models. First, for a subset of ARC data, properties of the static propensity of ticket cancellations are explored using a multinomial logit model (MNL). Second, based on the occurrence of refund and exchange events in sample of ticketing data from the Airline Reporting Corporation (ARC), a DTPO "cancellation" model for the outbound legs of an airline itinerary for groups ticketing within 90 days of flight departure is estimated. Third, the goodness of the DTPO model is compared with a series of binary logits using ROC analysis.

In contrast to current state of practice for cancellation models which considers cancellation probabilities to be memoryless, we show that the propensity of cancelling develops from a combination of time effects. Our findings reinforce latest empirical studies in the airline industry and indicate that the intensity of the cancellations process depends both on days from departure and days from issue. Moreover, higher cancellations rates are observed for recently purchased tickets, and for tickets whose associated flight departure dates are near.

In addition to time effects, this dissertation demonstrates the dependence of the cancellation process on several other covariates. As expected, segmentation effects (*i.e.*, Saturday night stay) are significant with leisure passengers less prone to cancel than business passengers. Similarly, compared with itineraries departing late days of the week (Thursday, Friday, and Saturday) the intensity of cancellations of itineraries departing early days of the week (Monday, Tuesday, and Wednesday) is higher. Finally, higher groups have lower cancellation rates.

Fourth, to assess the effect of time-to-event forecasts on revenue stream a complete single resource capacity control (SRCC) simulation is designed and executed. Results of the simulation indicate that compared with current state of practice of cancellation forecast (*i.e.* bookings on hand and anchored in departure date), time to event cancellation forecast result in additional revenues up to 9%.

One of the limitations of current dissertation is that it reports a smaller proportion of cancellations (less than 8% across all markets) than proportions from booking data. For example, Smith, Leimkuhler et al. (1992) report a combined no-show and cancellation proportion of 50% for American Airlines; while these rates vary across carriers and markets and may have decreased over time, cancellation proportions of 30% or more are not uncommon today.

Therefore, one of the questions that naturally arise from this study is: Why is there a large discrepancy in cancellation proportions between booking and ticketing data? One possible explanation is that booking data (and revenue management systems) are capturing the initial searching and pre-purchasing behavior of passengers. This would occur, for example, if a business traveler called a travel agent to booked a reservation, but then waited a few days to either modify or pay for the reservation once travel plans became more firm. In general, the time period a reservation can be held is short – 24 to 48 hours. Thus, failure to pay for a reservation could lead to rebooking the same (or similar) itinerary multiple times. Part of this booking activity or booking "churn" as it is more commonly referred to may be represented in ARC "void" data. The void data represents tickets that were created, but not purchased and thus "voided" before a financial transaction was required.

Acknowledging this statute of limitation current dissertation uses a range of cancellation percentages (10% to 30%) to "inflate" the original cancellation percentage, to determine the impact of time to event forecasts. In the context of missing cancellations (voids, other), this adjustment should provide good estimates provided that the effect of the booking churn is constant over time.

As such, one possible extension of current analysis is to estimate a model that will estimate and possible isolate the effect of cancellations from voids. This should permit a better linkage between cancellation rates determined using ARC ticketing data and cancellation rates determined using booking data. Also, if we assume that the risk of canceling tickets due to voids is fundamentally different from the risk of cancelling purchased tickets (with the former having a much smaller probability of surviving past two days), other modeling methodologies (including competing risks or a multi-stage estimation approach) may be appropriate.

The second area of further refinements is way in which cancellation forecasts are implemented in the SRCC simulation. Although, transitional properties of the cancellation process are estimated using disaggregate data, the SRCC simulation uses estimated average effects when forecasting for cancellations. Despite its tractability in the context of traditional revenue management, the SRCC simulation does not assess the impact of time to event forecasts for a discrete choice revenue management. To fully capture the effects of a time-to-event cancellation forecast at a disaggregate level the SRCC simulation needs to be adjusted accordingly. In this context, a new algorithm capable of jointly optimizing capacity allocation and overbooking controls in the context of non-stationary cancellation process needs to be designed.

Appendix A - Airline Reporting Corporation (ARC) fields

TERM	DEFINITION	POSSIBLE VALUES
1. Carrier	The airline reported on the flight coupon as the transporting carrier for an airport pair.	ARC assigns a random number to identify unique carriers within the specified city pair, yet mask the identity of an individual carrier.
2. Issue Date	The date the specified ticket was issued by the ARC	Any day, formatted as MM/DD/YY, prior to and including the Departure Date.
3-4. Departure Date	The departure date for inbound and outbound segments	Any day in the calendar year 2004, formatted as MM/DD/YY.
5-6. Departure Date New	The departure date for inbound and outbound segments, displayed in association with an exchange event	Any day, formatted as MM/DD/YY, prior to, including, or after the Departure Date (original).
7. Exchange Date	The date, as applicable, associated with an exchange event - full or partial - on a ticket.	Any day, formatted as MM/DD/YY, prior to, including, or after the Departure Date (original);
8. Refund Date	The date, as applicable, associated with a refund event on a ticket.	Any day, formatted as MM/DD/YY, prior to, including, or after the Departure Date (original);
9. Void Date	The date, as applicable, associated with a voidevent on a ticket.	Any day, formatted as MM/DD/YY, prior to, including, or after the Departure Date (original);
10. Exchange Fee	The fee, if any, associated with an exchange event.	A numeric value, expressed in USD currency, formatted to the hundredths (e.g. $49.75).
11. Fare	The value, net of taxes and other fees, listed on the 'Fare' field of a ticket.	Any numeric value, expressed in USD currency, greater than or equal to 0, formatted to the hundredths (e.g. $209.99)
12. Fare Difference	Exchange event = calculated by subtracting the old fare value from the new fare.	A positive or negative numeric value, expressed in USD currency, formatted to the hundredths (e.g. $49.75).
13-14. New Flight # Ind	A code applied to indicate when the flight number associated with an exchange event is different	A 1-character numeric value. 1 = Yes; 0 = No;
15. Ticketing Class	A high-level categorization of tickets associated with the first character (prime code) on the Fare Basis Code;	First, Business, Economy/Coach, or Other. First = (A, F, or P); Business = (C, D, I, J, or Z); Economy/Coach = (B, H, K, L, M, N, Q, S, T, V, W, X, or Y); Other = (E, G, O, R, or U).
16. Ticketing Class New	If for an exchange event the value is different from the original value.	First, Business, Economy/Coach, or Other.
17-18. Ticketing Class Code	The first character (prime code) of the Fare Basis Code, defined by carriers to specify the type of fare applicable. (e.g. F,	A 1-character alpha value.
19-20. Ticketing Class Code New	The first character (prime code) of the Fare Basis Code, reported in association with an exchange event	A 1-character alpha value.
21. Trip Type	The type of trip on an itinerary.	A 2-character alpha value; OW (One Way) or RT (Round Trip)

Appendix B - ROC Analysis Estimation Results

	HAZARD	LOGIT 0_3	LOGIT 4_7	LOGIT 8_14	LOGIT 15_21	LOGIT 22_45	LOGIT 46_90
DFI_0_3	0.573***						
DFI_4_7	1.116***						
DFI_8_14	0.943***						
DFI_15_21	0.763***						
DFI_22_45	0.551***						
DFD	-0.0378***						
GSize2	-0.816***	-1.149***	-1.075***	-0.890***	-0.819***	-0.772***	-1.050***
GSize3plus	-1.235***	-1.079**	-1.227***	-1.367***	-1.595***	-1.315***	-1.409**
SatInd	-0.265***	-0.324*	-0.470***	-0.387***	-0.549***	-0.0865	-0.335
Monday	0.244***	0.0800	0.340*	0.631***	0.435*	0.166	0.0557
Tuesday	0.252***	0.117	0.454**	0.629***	0.392*	-0.101	0.301
Wednesday	0.0923	-0.242	0.354*	0.470**	0.126	-0.111	0.132
Thursday	-0.107	-0.420*	0.0453	0.188	0.325	-0.606***	0.505
Friday	-0.178*	-0.103	-0.282	0.175	-0.161	-0.407*	-0.247
Saturday	-0.0269	-0.206	-0.0903	0.247	0.0972	-0.227	0.172
BosSea	-0.426***	-0.186	-0.425**	-0.446**	-0.722***	-0.678***	0.0775
HnlOrd	-0.983***	-1.423*	-0.425	-1.484**	-1.507**	-1.500***	-2.363*
MiaBos	-0.532***	-0.597**	-0.431**	-0.240	-0.410*	-0.686***	-1.472*
MiaSea	0.343***	0.105	0.356	0.599***	0.0212	0.319	-0.430
OrdHnl	-0.381***	-0.852**	-0.859***	-0.650**	-0.443	-0.457*	-1.235*
SeaBos	-0.416***	-0.243	-0.483**	-0.352*	-0.419*	-0.697***	-0.453
SeaMia	-0.505***	-0.407	-0.737***	-0.468*	-0.912***	-0.331	-1.057*
Carrier2	0.138*	0.0163	0.181	0.121	0.237	0.203	0.0239
Carrier3	-0.981***	-1.152***	-0.749***	-0.968***	-1.371***	-0.819***	-1.487**
Carrier4	-0.252*	-0.455	-0.199	-0.274	0.0500	-0.243	-0.382
Carrier5	0.094	0.0623	0.199	-0.249	0.188	0.261	0.454
FarePro	0.00127***	0.00119***	0.00162***	0.00193***	0.00185***	0.00205***	0.00236***
_cons	-6.813***	-5.238***	-5.047***	-5.002***	-4.837***	-4.082***	-4.501***
N	2894317	113551	100941	86011	67758	52018	17205
pseudo R-sq	0.057	0.038	0.053	0.052	0.051	0.045	0.065
LL 0	-20148.6	-2545.3	-3326.5	-3512.5	-2255.0	-2926.0	-700.5
LL	-18991.3	-2447.9	-3151.7	-3328.9	-2141.1	-2795.8	-654.9

* p<0.05 ** p<0.01 *** p<0.001

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